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Real options models and analyses for energy transportation systems with jump and diffusion processes: Electric power transmission planning and fuel-carrying ship design

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Real options models and analyses for energy transportation systems with jump and diffusion processes: Electric power transmission planning and fuel-carrying ship design

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ABSTRACT

Investments in transportation sector have arisen as significant problems due to the substantial cost of maintaining required level of service. Energy transportation draws a particular attention. In this dissertation, we focus on two special forms of energy transportation problems. On the one hand, we study the quantification of value of expansion investments in high-voltage power lines. On the other hand, we quantify the value of expansion of capacities for U.S. Navy transportation ships. Problems are subject to severe uncertainties being in the form of smooth changes and discrete disruptions. We use geometric Brownian motion and Poisson arrival processes to model both types of evolutions, respectively. We utilize real options approach to quantify the values of expansion options and provide decision makers with managerial insights.

My dissertation consists of three papers. In the first paper, we quantify the value of expansion options in transmission lines. Revenue of the investment is calculated based on differences between locational marginal prices. This research reveals that the proportion of susceptance of a power line to its power-carrying capacity is a significant factor to determine the value of an expansion investment. In the second paper, we quantify the value of option to expand the capacity of a U.S. Navy transportation ship. Decision maker can either choose flexible design (relatively more prepared for expansion) or fixed design at initial design phase. Our study indicates that flexible design should be preferred over fixed design if initial demand (for transported item) value is relatively lower. Third paper revisits transmission expansion problem and adopts installation of local generators as an uncertainty in the form of discrete disruption. It shows that the value of transmission network does not always reduce with the installation, instead location of the installation plays a key role.

CHAPTER 1. GENERAL INTRODUCTION

Engineering is a discipline, which shows the best ways of fulfilling tasks and of reaching to desired goals. Through executing these tasks, there exist many tactical and operational level decisions that should be made by the decision makers. These decisions in fact represent engineering problems from which they are resulted. Each problem has an objective to be reached by the decision maker, but there exist other significant parts of these problems: Constraints based on physical laws and principles, which serve as the fundamental basis of engineering. Engineering problems arise with constraints, which should be obeyed while resolving to the problems.

Of engineering problems with critical constraints, those in transportation area draw attentions of research practitioners and industry professionals because of huge costs of maintaining the qualified service and significant uncertainties. To stay competitive in challenging business environments, the transportation companies pay attention to their tactical level decisions. They often face real-life problems that should be solved with real-life constraints.

Set of transportation problems consist of various types. The importance of these problems can be evaluated by the items transported. Energy transportation is precisely one of the most crucial areas in which critical decision making ought to be followed. By 2040, it is estimated that transportation energy demand will increase to nearly 75 million barrels of oil per day (Burns 2013). Electric power transmission and fuel transportation arise as two significant subsets of energy transportation problems. Both energy sources, electric power and fuel, will likely exist to sustain the human life forever, which make them indispensable in this respect. That is why many real-life engineering problems with relevant constraints are defined and constructed surrounding those energy sources.

In this dissertation, specific problems arising in electric power transmission and fuel transportation areas are considered and modeled with their physical constraints. Specifically, electrical circuit laws, known as Kirchhoff current and voltages laws, are taken into account as constraints in electric power transmission. As for fuel transportation, the relationship between speed, power, length and total mass of transportation vehicles represents constraints. The main objective of this dissertation for both problems is to show how economic decisions subject to these constraints can be made under long-term uncertainties.

In these two areas, the problems are dynamic and subject to uncertainties. We use stochastic optimal control techniques to derive the managerial insights to be presented to the decision makers because the pattern of demand is often modeled as geometric Brownian motion (GBM). Among various stochastic optimal control techniques, we utilize real options approach to model these problems. The distinguishing part of this approach is to quantify and add the value of existing managerial flexibilities into the investment value derived from well-known net present value (NPV) approach.

Let us examine in detail two main problems as follows. In Chapter 2, a real options framework that provides the valuation of a transmission owner's option to expand in his or her network is developed and analyzed. What distinguishes this framework from the extant literature is that the evolution of the demand follows GBM process, it explicitly accounts for the physical flow of the electric power economically manifested as the locational marginal prices (LMP), and it shows how the values of the expansion options can be determined in the transmission network. Furthermore, this framework shows how to value an option to expedite or delay can be determined given that a specific expansion is planned. It reveals that the proportion of susceptance (measures the easiness of electric power flow on a transmission line) of any transmission line to its power carrying capacity is influential factor to determine the value of the investment in the corresponding circuit. An extensive numerical example is presented to illustrate the key features of this framework. A compact version of this chapter is published in *The Engineering Economics* journal.

In Chapter 3, an engineering design problem arising in fuel transportation of the navy is concerned. In the current environment of constrained expenditure often subject to budget cut, when transportation requirements increase in the navy, then some of the practical solutions involve jumboization. Jumboization is defined as increasing the capacity of an existing ship by extending its length at a future date. In view of this jumboization, the choice of the ship design has future ramifications as follows. With fixed design (the ship is not designed initially envisioning future jumboization investment), jumboization later will be costly. With flexible design (the ship is designed initially envisioning future jumboization investment), jumboization later will be less costly, however the initial cost may be more because of initially strengthened hull of the ship. In this chapter, we construct and analyze both cases, and determine conditions under which one design is superior to another by using stochastic optimal control approach. Jumboization results in fuel cost saving per ton due to the decrease in wave-making resistance of the ship. For engineers, managers and/or military officers who make decisions, we show that relatively low level of initial transportation requirement is a signal for the decision maker to prefer fixed design subject to relationships between displacement, speed, length of the ship and power. Moreover, longer distances the ships are required to transport are in favor of jumboizing the ships earlier. Key components of our framework are illustrated with a numerical example based on a real ship. A compact version of this chapter is submitted to *Systems Engineering* journal.

We now extend Chapters 2 and 3 in major way as follows. Chapters 2 and 3 consider smooth changes of uncertain parameters in their context. Decision makers of private and public sectors frequently face with smooth changes of uncertainties while making strategic decisions. However, decision makers have also encountered several types of unexpected and large-scale fluctuations (discrete disruptions), which have often catastrophic ramifications such as so-called black swan events (rare events) in financial sector. There is a need for a framework to model the evolutions of smooth changes and discrete disruptions so that investments (in both financial and real sectors) can be evaluated. In Chapter 4, we develop a new computationally efficient lattice method to model both types of uncertainties and apply it to transmission expansion investments. In recent years, decision makers of expansion investments have faced critical uncertainties such as growth of demand for electricity and installation or removal of distributed generations (DGs), which are local generations for small communities. This circumstance indicates that expansion decisions for transmission lines should be made strategically as installation of DGs may create a stranded cost for transmission owners because DGs meet a portion of local electricity demand. In Chapter 4, we propose a real options framework which quantifies the value of expansion investments under demand and DG uncertainties. The treatment of uncertain parameters is achieved by using GBM combined with compound Poisson process. Proposed framework is demonstrated on a hypothetical example to illustrate the key components of our framework. It shows decision makers should not unquestionably think that DG installation in one area decreases the value of transmission network. Instead, they should focus on the location of DG installation as it is a significant factor determining the trend in the value of transmission network.

We can count three commonalities in Chapter 2, Chapter 3, and Chapter 4. First, the type of real option evaluated in three chapters is expansion option. For transmission networks, we consider that decision maker has a right, but not obligation, to expand the power network by installing a power line between centers. For ship design, we think that decision maker has a right, but not obligation, to expand the capacity of a replenishment oiler by extending its length. A second commonality is that problems of our interests arise in energy transportation sector. Transmission networks transfer electric power and replenishment oilers carry fuel, both of which are special types of energy commodities. Lastly, from the methodological perspective, we use the same approach to model the investment problems in transmission networks and ship design. We use real options approach, or stochastic optimal control framework, to model problems and to quantify the values of investments in both application areas.

The structure of this dissertation is as follows. In Chapter 2, expansion investments are evaluated with real options approach in electric power transmission network. In Chapter 3, jumboization of a military transportation ship is considered as a real option for the decision maker and is quantified to resolve the comparison problem of flexible and fixed designs. In Chapter 4, the way of how transmission investments are evaluated under demand and DG installations uncertainties are shown. Chapter 5 is an overall appendix, which lists a couple of discussions related to Chapter 2 and Chapter 3. Chapter 6 concludes this dissertation by emphasizing significant parts and summarizing major findings.

CHAPTER 2. EXPANSION PLANNING FOR TRANSMISSION NETWORK UNDER DEMAND UNCERTAINTY: A REAL OPTIONS FRAMEWORK

Introduction

Since the deregulation of U.S. electric power in the 1990s, the transmission aspect of the electric power industry has been separated from the generation aspect, and the responsibilities of the transmission network owners have been much different from the responsibilities of generation unit decision makers (we will use owners and decision makers interchangeably because the decisions made in this article are on behalf of the owners). For example, many generation unit decision makers have no obligation to serve, whereas transmission owners are expected to address increasing demands and still maintain technical requirements such as reliability and stability. For this reason, there have been numerous sophisticated studies on transmission expansion planning (see, e.g., Buygi et al. 2004), which are characterized by uncertainties ranging from demands to fuel costs, substantial and upfront expansion investment costs, and irreversibility of the expansion investment.

In the often-practiced case of the hybrid merchant/regulated mechanism for the expansion investment, a major part of the revenue needed for expansion is collected from the market participants such as distribution utilities and power generators. For instance, in California, participating transmission owners, who obey the regulatory authority of the independent system operator, are allowed to collect the transmission access charge (California ISO 2014a, 2014b). The other major part is through the financial transmission rights, which depend on the LMP differences. In this mechanism, the transmission network owners hold financial transmission rights and sell them to market participants to generate the other major part of the revenue needed for expansion (see, e.g., Pringles et al. 2014).

We note that, from the perspective of numerous transmission owners in the hybrid merchant/regulated mechanisms, the expansion (and when to do it given that they would do it) can be viewed as strategic real options offering managerial flexibility under uncertainties (see, e.g., Dixit and Pindyck 1994).

In this article, for such owners in the hybrid merchant/regulated mechanisms, we show how the values of the expansion options can be determined in the transmission network. Furthermore, our framework shows how to evaluate an option to expedite or delay given that a specific expansion is planned. This is achieved under the assumption that the evolution of the demand follows a GBM process and there are no other uncertainties, and through the optimal power flow (OPF) calculations leading to the appropriate LMP levels.

The rest of the chapter is organized as follows. We first present a review of the relevant literature. Next, we explain the general framework of our chapter. This is followed by an extensive numerical example that illustrates the key features of our framework. Finally, we make concluding remarks and provide technical appendices on the OPF formulation and the LMP calculations.

Literature Review

For the electricity market, the real options approach has frequently been studied for generation expansion planning. As for transmission expansion planning, the real options approach has been less frequently studied. Of such studies on transmission expansion planning, there are primarily three groups of real options applications.

In the first group, the configuration of the transmission network is simply bi-nodal (a network of two nodes). Hence, there is only one investment decision under consideration (i.e., to add a power line between two nodes; see, e.g., Abadie and Chamorro 2011).

The second group of studies investigates the option to defer the transmission investment. In this case, one can separate such studies into two subgroups based on their network configurations. In one subgroup, the researchers quantify the option to defer in bi-nodal networks (see, e.g., Blanco et al. 2009). In the other subgroup, the researchers quantify the option to defer in multi-node networks of three or more nodes (see, e.g., Osthues et al. 2014).

In the third group, there exist studies focusing on special electrical devices such as flexible alternating current (AC) transmission systems (FACTS) and DGs. Blanco et al. (2011) evaluated the transmission investment in FACTS devices. In the model, the transmission owner invests either in a transmission line or in FACTS devices that relieve the transmission congestion. Similarly, Vásquez and Olsina (2007) focused on the deferral effect of DGs in transmission investments. The authors claimed that the owner can postpone the investment in a transmission network by constructing DG units that relieve the transmission congestion.

General Framework

In this section, we will first elaborate the revenue being generated by the LMP differences and then address the lattice construction for demand uncertainties. This is followed by the investment valuation process with flowcharts. We note that the brief information regarding the OPF formulation and the way of calculation of the LMPs is provided in Appendices 2.A and 2.B, respectively.

Revenue Generated by the LMP Differences

When generation centers (nodes in network) are dispatched at optimality of the OPF problem and if they are paid according to their own LMPs and the consumption centers (other nodes in the network) pay for electricity according to their own LMPs, then there exists a surplus amount of money (see, e.g., Hsu 1997; Pérez-Arriaga et al. 2013). This surplus results from the congestion in the network and it is generally accepted as the source of revenue for the network owner (see, e.g., Garcia et al. 2010; Pringles et al. 2014). This revenue is modeled as in Equation (2.1)

$$\sum_{i \in N_D} \pi_i D_i - \sum_{j \in N_G} \pi_j G_j \tag{2.1}$$

where π_i denotes the LMP at center i, D_i denotes the demand amount at center i, G_j denotes the dispacthed amount of power from generator at center j (at optimality of the OPF problem), and N_D and N_G denote the set of consumption centers and generation centers, respectively. Krause (2003) stated that Equation (2.1) is always larger than zero if at least one transmission line is congested. If none of the power lines (arcs in network) is congested, then the difference equals zero. Although we cannot present the details of this payment mechanism (for details of such a mechanism, see, e.g., Kirschen and Strbac 2004), in this article, we utilize a simpler version of this difference as the revenue of the network. We note that this revenue is on an hourly basis (the unit is dollars per hour) because the unit of the LMPs is dollars per megawatt-hour and the units of D_i and G_j are megawatts.

We note that only the transmission income is considered in this context because we make an attempt to solve the problem of the transmission owner. As the above discussion implies, the income of the transmission owner results from differences between the LMPs. In the literature, various studies can be found that take into account only the transmission income to solve the investment decision problems.

For example, Pringles et al. (2014) examined the impact of fixed revenue provided by the regulatory authority on the investment decisions made by the transmission owners. The authors defined the revenue for the transmission owner as consisting of only two parts: variable revenue and fixed revenue. Whereas the former one represents differences between the LMPs, fixed revenue is paid to the transmission investor by the regulatory authority, as we adopt in our study. In another study, transmission investments were evaluated by Garcia et al. (2010) to find the optimal time of the investments. The authors indicated that investments in transmission assets are only remunerated by differences between the LMPs. Having calculated the revenue in each year, a NPV curve was created in order to reveal the optimal investment time. In Blanco et al. (2009), the option to defer in transmission investments was evaluated. The authors assumed that the revenue of an investment arises as a result of differences between the LMPs. A different study was conducted by Fu et al. (2006), who considered two types of behavior of transmission investors. Whereas one attempts to minimize the investment cost, the other pursues maximizing the revenue of the investment. In the latter one, the revenue of the investment is assumed to be generated only from differences between the LMPs. Finally, Ramanathan and Varadan (2006) introduced an overview of a modeling framework to evaluate the transmission investments under uncertainties with the real options methodology. Differences between the LMPs were put forward by the authors as a single source of revenue of the investments.

Uncertainty and Discretization by the Lattices

Because option evaluation based on a continuous stochastic process such as GBM is difficult, we intend to use the discretized form of this process. Before describing demand growth modeling, we focus on a demand evolution in a single consumption center in order to present the binomial lattice discretization more clearly. Then we introduce the multiple branch lattices to illustrate the demand growths in multiple consumption centers.

We note that our binomial lattice approach has a computationally weak point, especially with a multiple number of underlying uncertainties (namely, the curse of dimensionality; see, e.g., Abadie and Chamorro 2013; Andersen and Broadie 2004).

On the other hand, the binomial lattice approach has been successfully utilized as a modeling approach of underlying uncertainties and its usefulness has been mentioned extensively in the literature. It is well known that when continuous stochastic processes are used, many options (and real options) problems lead to intractable solutions. The reason is that valuation functions mostly turn out to be partial differential equations and they can rarely be solved in closed-form solution. Therefore, discrete models have to be developed and implemented in order to obtain a solution (see, e.g., Pacheco and Vellasco 2009).

Several discrete models have been proposed in the literature as alternatives to continuous models. For instance, Brennan and Schwartz (1978) developed well-known implicit and explicit methods for valuing options. Among the discrete models, the binomial lattice has been one of the most frequently used models (see, e.g., Hull 2009). It is stated that the binomial lattice is highly flexible to incorporate complex real options and it is easy to implement (see, e.g., Mun 2002). Moreover, it allows pricing American options, which is a required property in the real options area because most of the real options can be exercised prior to their maturity. Luenberger (1997) supports the idea that many real-life problems can be solved with the binomial lattice. Mathematical properties of the binomial lattice are other reasons why it is usually preferred. For example, lognormal distribution of the evolution of asset values can be well approximated by the binomial lattice. It also allows incorporating a risk-neutrality property, which is a strong assumption in real option valuations.

Additionally, the power of the binomial lattice in modeling has been mentioned in the literature. For example, Boyle (1988) established a lattice model to represent two underlying state variables. He verified the accuracy of the developed model by evaluating European options. He compared the values of European options derived from his lattice model with the accurate values published in other studies. He revealed that the differences are not significant and thus concluded that the developed lattice framework can be securely used for the most applications.

Single consumption center

In this section, by taking into account the uncertain demand growth in a consumption center, we discuss the derivation of the binomial lattice parameters, discount rates, and the importance and derivation of risk-neutral probability. We will address more complicated uncertainty discretization in multiple consumption centers afterwards.

Derivation of parameters for the binomial lattice

One of the most commonly used discretization methods is the binomial lattice developed by Cox et al. (1979). According to this method, a variable X (in our case, X represents D_i, where i is the single consumption center) has two possibilities after one period; it either goes up or goes down. The change in X is determined by the multiplication factors $u > 1$ and $d < 1$. In other words, it becomes either uX or dX with probabilities p and $1 - p$, respectively (Figure 2.1). Therefore, mathematical expressions of the parameters u, d, and p should be determined.

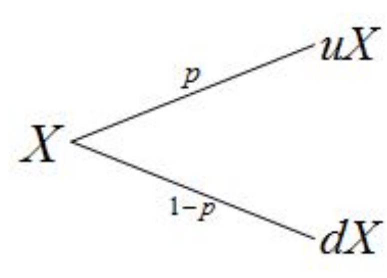

Figure 2.1 *One-step lattice*

Discretization of the GBM process can be performed by considering the natural logarithm of the change in X, which is denoted as $\ln X$. The binomial lattice matches the expected value and the variance of $\ln X$. By following the derivation procedure shown by Luenberger (1997), the parameters can be obtained as

$$u = e^{\sigma\sqrt{\Delta t}} \tag{2.2}$$

$$d = e^{-\sigma\sqrt{\Delta t}} \tag{2.3}$$

$$p = \frac{1}{2} + \frac{1}{2}\left(\frac{\mu - \frac{1}{2}\sigma^2}{\sigma}\right)\sqrt{\Delta t} \tag{2.4}$$

where μ is the drift parameter of the process X, σ is the volatility of the process X, and Δt is the length of one time period in the lattice. We note that Δt designates a degree of time granularity ranging from days to perhaps years. We also remark that the probability p is derived from the discretization of the GBM process; hence, it is not a risk-neutral probability. In the binomial lattice calculations, risk-neutral probability should be used instead of p.

Discount rates

A discount rate is the interest rate used in discounted cash flow analysis to calculate the present value of the future cash flows. Discount rate takes into account not only time value of the money but also risk included in future cash flows (Investopedia 2014).

If it is not desired to include the risk, it is viable to utilize the risk-free discount rate. Zacks Investment Research (2014) states that the risk-free discount rate is typically the amount that an owner expects to gain from an investment in a zero-risk security. In general, the yield on a U.S. Government bond is accepted as risk-free discount rate.

In the context of company businesses, different discount rates are used to evaluate the projects because they have risk. According to Investopedia (2014), the weighted average cost of capital is generally used when the project risk profile is similar to the company's risk profile. On the other hand, if they are different, a capital asset pricing model is frequently used to determine the project-specific discount rate. The discount rate calculated in this way is called the risk-adjusted discount rate.

The risk-adjusted discount rate is defined as the sum of the risk-free discount rate and risk premium (see, e.g., Investopedia 2014). Risk premium can be calculated as (Market rate of return - risk-free discount rate) multiplied by the beta of the project. More specifically, Investopedia (2014) notes that the beta of the project represents the extent of "how much a company's share price moves against the market as a whole" (paragraph 9). If beta is equal to one, then they move in line with each other. Otherwise, if it is larger than one, the share is said to exaggerate the movements of market, and if it is less than one, it is said to be more stable.

Risk-neutral probability

In the binomial lattice calculations, risk has to be included in the equations. Mun (2002) states that cash flows including risk must be adjusted so that risk can be represented. According to Mun (2002), there exist two methods for doing this: (i) cash flows are calculated by utilizing the risk-adjusted discount rate or (ii) probabilities of the cash flows are adjusted with risk and discount of cash flows is performed with risk-free discount rate. Though original (or true) probabilities are taken into account in calculations for (i), risk-neutral probabilities are considered in calculations for (ii). The methods defined in (ii) are preferred in real options analysis because it is expressed that calculating different risk-adjusted discount rates in various states through the binomial lattice is avoided in this case. The following simple example depicted in Figure 2.2 (Mun 2002) explains how risk-neutral probability is obtained.

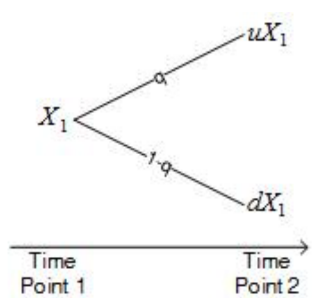

Figure 2.2 *Simple payoff*

Let X_1 be the payoff of a game. The expected payoff at time point 1 (X_1) is simply calculated as

$$X_1 = (quX_1 + (1-q)dX_1) \cdot (1+r)^{-1} \tag{2.5}$$

where r is the risk-free discount rate. By assuming that $X_1 = 1$, then

$$1 = (qu + (1-q)d) \cdot (1+r)^{-1} \tag{2.6}$$

gives the risk-neutral probability for up movement as

$$q = \frac{1+r-d}{u-d} \tag{2.7}$$

where u and d are calculated by using Equations (2.2) and (2.3), respectively. Alternatively, in Wang and Min (2006), it is stated that risk-neutral probability can be derived by replacing μ in Equation (2.4) with r.

After these discussions, we can now proceed to present a more general case of multiple consumption centers.

Multiple consumption centers

If demand growth is an uncertain factor in multiple consumption centers, then the binomial lattice turns into the multiple branch lattice because demand in consumption center i, D_i, has different drift and volatility parameters than those of demand in consumption center j, D_j. A state in the lattice (denoted by (t,k) where t denotes the time point and k denotes for each t vertical numbering starting with 1 from the uppermost state and increments through the undermost state) consists of the demand vector $(D_1, D_2, \dots D_{|N_D|})$, where N_D is the set of consumption centers. Hence, the number of branches (arcs in the lattice) emanating from a state in the lattice is $2^{|N_D|}$. In order to calculate the demands of the next time point's states in the lattice, all u_i and d_i possibilities are taken into account in a permutational manner.

Figure 2.3 illustrates an example of demand evolution in two consumption centers for two periods. We note that period a is defined from time point a to time point $a+1$.

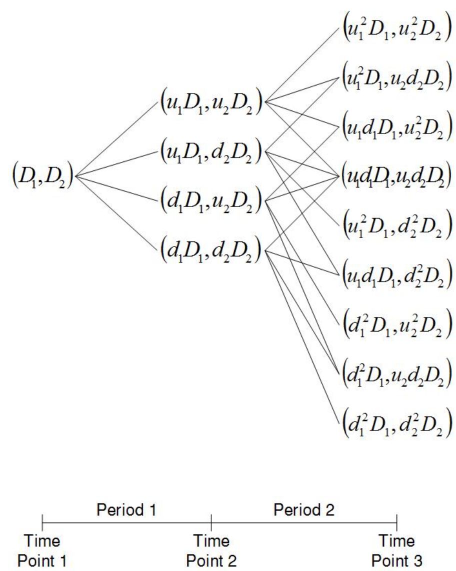

Figure 2.3 *The multiple-branch lattice*

Although finding the permutation of all u_i and d_i at the beginning of the next period is easy, it is not trivial to come up with the probabilities of the branches in the multiple branch lattices similar to Equation (2.4). According to Wang and Min (2006), if there is no correlation between the demand growths, then the joint probabilities of the branches can be found with the multiplication of marginal probabilities. If there is a correlation between demand growths, then

$$p_l = \prod_{i=1}^{|N_D|} p_{\delta_i(l)} + \frac{1}{2^{|N_D|}} \sum_{i=1}^{|N_D|} \sum_{j=i+1}^{|N_D|} \delta_{ij}(l)\rho_{ij}\,, \quad l = 1,2,\dots,2^{|N_D|} \tag{2.8}$$

gives the joint probability for branch l, where ρ_{ij} is the correlation coefficient between demand growths in consumption centers i and j. Moreover, $\delta_i(l)$ and $\delta_{ij}(l)$ are defined as follows:

$$\delta_i(l) = \begin{cases} u_i, & \textit{if} \text{ demand in center } i \text{ has up movement in branch } l \\ d_i, & \textit{if} \text{ demand in center } i \text{ has down movement in branch } l \end{cases} \tag{2.9}$$

$$\delta_{ij}(l) = \begin{cases} 1, & \textit{if} \text{ demands in } i \text{ and } j \text{ move in the same direction in branch } l \\ -1, & \textit{if} \text{ demands in } i \text{ and } j \text{ move in the opposite directions in branch } l \end{cases} \tag{2.10}$$

We note that in Equation (2.8), p_{u_i} is the probability defined in Equation (2.4). Since we need risk-neutral probabilities, we first convert p_{u_i} to the risk-neutral probability q_{u_i} by replacing μ_i in Equation (2.4) with r. Then, if we use q_{u_i} instead of p_{u_i} in Equation (2.8), we obtain risk-neutral probability of branch l.

Investment Valuation Process

Before elaborating details of transmission investments evaluation, we present the notations frequently used during creation of the evaluation lattices in Table 2.1 in order to facilitate the understanding of the critical flowcharts in this subsection.

Given the notations in Table 2.1, the general process for the investment valuation is as follows. For a given network, an investment alternative is defined as adding a power line between two selected centers. In the valuation approach, the investment alternatives between each pair of centers are evaluated separately.

We rely on the NPV lattice without investment for the procedure to evaluate each investment alternative. Therefore, the way of creation of the NPV lattice without investment is presented first.

Table 2.1 *Notations for the investment valuation process*

Notation	Explanation
t	A time point in the multiple branch lattice
T	Last time point
k	Vertical numbering of the states in the lattices. For each t, its value starts with 1 at the uppermost state and increments through the lowermost state. For example, in Figure 2.3, k starts with 1 from above, increments through bottom, and becomes equal to 9 at the lowermost state for $t = 3$.
$E_{(t,k)}$	Demand vector at the state (t, k) in the demand evolution lattice
$S_{(t,k)}$	Set of all successor states of (t, k) at the next time point $t + 1$. For instance, the cardinality of $S_{(t,k)}$ is 4 in Figure 2.3.
$B_{(t,k)}$	Set of all branches that directly emanate from (t, k). The cardinality of $B_{(t,k)}$ is 4 in Figure 2.3.
q_l	Risk-neutral probability of the branch l, $l \in B_{(t,k)}$
$NR_{(t,k)}$	The revenue per hour ($/h), which is the result of Equation (2.1), for a given demand vector $E_{(t,k)}$. The unit is dollars per hour because the unit of the LMP is dollars per megawatt-hour.
c	The fixed operation and maintenance cost ($/h)
r	Risk-free discount rate (%/year)
H	The number of hours in Δt
$NPV_{(t,k)}$	NPV of the gained total profit from time point t to time point $t + 1$ for (t, k) ($/$\Delta t$ years)
$V_{(t,k)}$	NPV of the network in the NPV lattice at time point t for (t, k) ($). Thus, as opposed to NPV_{tk}, it additionally includes the risk-neutral expected value of the successor states at the next time point. (see Equation (2.13))
$V_{(T,k)}$	NPV of the network in the NPV lattice at time point T for (t, k) ($)
DMC	Decommissioning cost of the network ($)
A	Supplementary revenue for the owner ($)
I	Initial investment cost ($)

Constructing the NPV lattice for the network without investment starts backwards. Thus, terminal state values $V_{(T,k)}$ should be determined first. At the terminal states, the LMP-based revenues and corresponding profits are calculated for Δt years. For the corresponding demand values at the state (T, k), we calculate the LMPs by solving the OPF problem. Then, by using Equation (2.1), network revenue denoted by $NR_{(T,k)}$ is computed and $NPV_{(T,k)}$ is calculated by using Equation (2.11) (Equation (2.11) is written with t to represent the general case because it is used for intermediate states as well). In addition to $NPV_{(T,k)}$ for the terminal states, we add the discounted decommissioning cost with Equation (2.12). Thus, we obtain $V_{(T,k)}$ for the terminal states.

$$NPV_{(t,k)} = H \cdot \left(NR_{(t,k)} - c\right) \cdot (1 + r)^{-\Delta t} \tag{2.11}$$

$$V_{(T,k)} = NPV_{(T,k)} + (-DMC) \cdot (1 + r)^{-\Delta t} \tag{2.12}$$

For the intermediate states, after calculating the corresponding $NPV_{(t,k)}$ with Equation (2.11), we add it to the risk-neutral expected value of the successor states of (t, k) (corresponding $V_{(t+1,k)}$) by using Equation (2.13). Thus, we find the NPV of the network at the present time denoted by $V_{(1,1)}$ by the recursive relation presented in Equation (2.13).

$$V_{(t,k)} = NPV_{(t,k)} + \left(\sum_{\substack{k \in S_{(t,k)} \\ l \in B_{(t,k)}}} q_l V_{(t+1,k)} \right) \cdot (1 + r)^{-\Delta t} \tag{2.13}$$

We now present the general flowchart, which is illustrated in Figure 2.4, for evaluating all investment alternatives existing in the network.

Figure 2.4

Box 1. In this step, an investment alternative such as adding a power line between centers 1 and 2 in the network is selected. The set of investment alternatives is defined as the collection of each investment alternative between a pair of centers in the network.

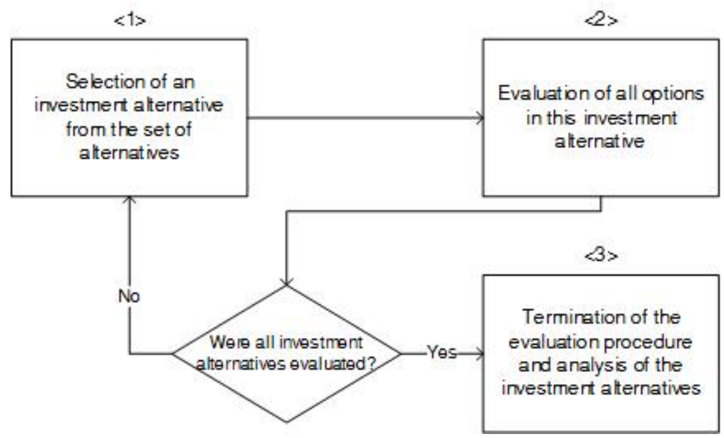

Figure 2.4 *Flowchart for investment alternatives evaluation*

Box 2. This procedure has a sub-procedure illustrated in Figure 2.5. Option t represents the investment made at the beginning of the period t. Therefore, for a model horizon equal to T, the owner has T options to evaluate. In each option, at the end of period T, a decommissioning cost is incurred. Moreover, we assume that transmission access charge A and initial investment cost I are incurred whenever an investment is made.

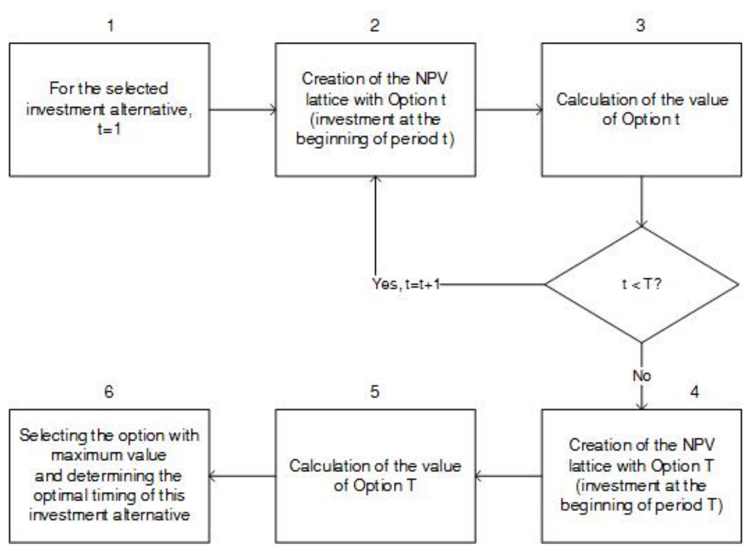

Figure 2.5 *Flowchart of evaluation of options existing in one investment alternative*

Figure 2.5

Box 2. Option t represents that an investment is made at the beginning of period $t < T$. Creating the lattice again starts with the terminal states and proceeds by backward induction. At the terminal states, we can still use the same equations, Equations (2.11) and (2.12). However, we note that because an investment is made before T, calculations of the LMPs and the LMP-based revenues ($NR_{(T,k)}$) are performed with respect to the new network configuration. For an intermediate state after t, we use the same equation, Equation (2.13), but we should add A and subtract I in Equation (2.13) at the beginning of period t because an investment is made at that time point. For states before t, we again utilize Equation (2.13), but we note that because an investment is not available at that time point, the LMPs and the LMP-based revenue calculations are performed by considering the network configuration without investment. Thus, with the recursive relations, $V_{(1,1)}$ is obtained with the investment made at the beginning of period t.

Box 3. The value of Option t is simply calculated as the difference between $V_{(1,1)}$ of the NPV lattice with Option t and $V_{(1,1)}$ of the NPV lattice without investment. If the latter one is larger than the former one, then we say that value of Option t is zero.

Box 4. Option T represents the situation in which an investment is made at the beginning of period T. In that case, at the terminal states, the owner still collects the revenue based on the LMP differences, represented by Equation (2.11). Because decommissioning cost is incurred at the end of period T, the corresponding cost should still be considered in Equation (2.12). However, A must be added and I must be subtracted in Equation (2.12) because the owner makes an investment at the beginning of period T. We note that, for Option T, the LMPs and the LMP- based revenue calculations are all performed with the upgraded network configuration at the terminal states. For the intermediate states, we can still use Equation (2.13), but network configuration without investment should be taken into account during calculations of the LMPs and the LMP-based revenues. Thus, by these recursive relations, we provide $V_{(1,1)}$ value at the present time with Option T.

Box 5. There does not exist any difference between methods in steps 3 and 5. In other words, the value of Option T is calculated as the difference between $V_{(1,1)}$ of the NPV lattice with Option T and $V_{(1,1)}$ of the NPV lattice without investment. If the former one is less than the latter one, then the value of Option T is said to be zero.

Box 6. In this step, values of all options are evaluated. Because it is better to have a larger value, the option with the maximum value is preferred. It also reveals the optimal timing of the investment.

Now, we turn to the upper procedure depicted in Box 3 of Figure 2.4 where investment alternatives are compared according to their optimal times and values.

Numerical Example

In this section, a small but comprehensive numerical example on a three-center network is presented. As can be seen in Figure 2.6, there are two generators at centers 1 and 2. The capacity of the first generation center ($\overline{G_1}$) is 100 MW and its generation cost (C_1) is \$40/MWh. The capacity of the second generation center ($\overline{G_2}$) is 200 MW and its generation cost (C_2) is \$30/MWh. We note that supply curves of these centers are assumed to be linear and not to change for the sake of simplification (see, e.g., California ISO 2005). In other words, generation center 1 is willing to produce each additional unit of electricity at \$40/MWh up to 100 MW and generation center 2 is willing to produce each additional unit of electricity at \$30/MWh up to 200 MW.

The capacities of the power lines ($\overline{L_{12}}$, $\overline{L_{13}}$, $\overline{L_{23}}$,) are 30, 36, and 35 MW, respectively. There is a consumption center at center 3 and the load amount (D_3) is 52 MW. Susceptance of the power lines is assumed to be equal.

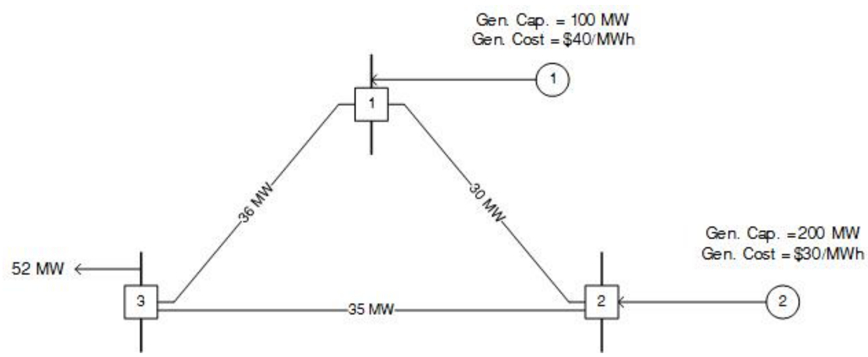

Figure 2.6 *Three-center example*

Because there exist two generation centers in adjacent places, it results in counter flow on the power line connecting centers 1 and 2 (for the details of this issue, please see Appendix 2.D showing the formation of the OPF formulation for the existing network). Thus, in this numerical example, we assess the impact of counter flow on profit and the value of the expansion option.

The OPF Problem

Throughout the numerical example, we solve the OPF problems by using the power flow equations analyzed by Bushnell and Stoft (1995). Because there are only two generation centers and one consumption center in the network, the equations proposed by Bushnell and Stoft (1995) can be utilized. Moreover, because those equations are more intuitive to understand the nature of power flows on the power lines in the case that two generation centers and one consumption center exist in the network, we prefer to switch from the classical OPF formulation to the formulation put forward by Bushnell and Stoft (1995).

In Bushnell and Stoft (1995), it is stated that network losses are negligible; voltage support and reactive power are not represented. Thus, linear power equations can be written by using the superposition theorem. This theorem says that net power amounts flowing on the lines can be found by considering only one generation center in each step. After finding the individual power flows triggered by only one generation center, net power flows can be found by adding these individual amounts algebraically. For details regarding how to construct the OPF problem analyzed by Bushnell and Stoft (1995), please see Appendix 2.D. We note that Appendix 2.A is different from Appendix 2.D in the sense that whereas the former one presents the classical OPF formulations, the latter shows the OPF formulation proposed by Bushnell and Stoft (1995).

The Demand Lattice

Because there is one consumption center in the network, it is legitimate to use the binomial lattice. We assume that the length of one period (Δt) in the binomial lattice is 1 year and the modeling horizon is 2 years. In Jin et al. (2011), drift and volatility of demand growth are estimated by analyzing real data from Midcontinent Independent System Operator website (MISO 2016). Drift (μ) and volatility (σ) are given as 0.0072 and 0.0094, respectively. However, for this numerical example, volatility was changed a bit in order to maintain consistency with other network parameters such as capacities of the power lines. Therefore, we use volatility equal to 0.13. We accept that initial demand is 52 MW. By using Equations (2.2) and (2.3), u and d values are calculated as 1.138 and 0.878. Thus, for demand evolution, the binomial lattice illustrated in Figure 2.7 is created.

Fifty-two megawatts in the demand lattice represents the beginning of the first period and 59.22 (or 45.66 MW) represents the beginning of the second period. We again note that (t, k) denotes the states in the binomial lattice and $E_{(t,k)}$ denotes the demand value at the state (t, k). Thus, $E_{(2,1)} = 59.22$, $E_{(2,2)} = 45.66$ and $E_{(1,1)} = 52$.

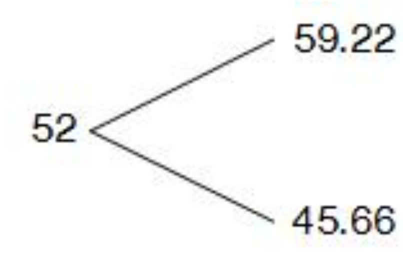

Figure 2.7 *The demand evolution lattice*

Investment Valuations

As can be seen in Figure 2.6, there are three investment alternatives: between centers 1 and 3, between centers 1 and 2, and between centers 2 and 3. In this section, we first create the NPV lattice for the network without investment. Then for each investment alternative, two options (investment made at the beginning of the first year and investment made at the beginning of the second year) are evaluated.

The NPV lattice without investment

The demand lattice triggers the NPV lattice without investment by matching each state of the demand lattice to the corresponding state of the NPV lattice. At the end of the second year, the network is removed and decommissioning cost is incurred. In this example, we assume that decommissioning cost of the existing network is $250,000.

We accept that fixed operation and maintenance cost (c) is $30/h and the risk-free discount rate (r) is 5%. Moreover, by using Equation (2.7) (it is enough to use Equation (2.7) instead of Equation (2.8) because we have just two branches that emanate from any state in the lattices) and u and d values equal to 1.138 and 0.878, respectively, we calculate the risk-neutral probability of up movement (q) as 0.66. Finally, we note that the number of hours in one year (H) is 8,760.

The results of the LMP calculations for all states and the network revenue per hour ($NR_{(t,k)}$) are given in Table 2.2. We note that the units of $E_{(t,k)}$ and G_i are megawatts, the unit of

π_i is dollars per megawatt-hour, and the unit of $NR_{(t,k)}$ is dollars per hour. Moreover, we remark that $NR_{(t,k)}$ is calculated as the difference between $\pi_3 E_{(t,k)}$ and $\pi_1 G_1 + \pi_2 G_2$ (see Equation (2.1). We note that we multiply π_3 with $E_{(t,k)}$ because $E_{(t,k)}$ is the demand at center 3) For the calculation details regarding the LMPs, please see Appendix 2.E.

Table 2.2 *LMP calculation - without investment*

t	k	$E_{(t,k)}$	π_1	π_2	π_3	G_1	G_2	$\pi_3 E_{(t,k)}$	$\pi_1 G_1 + \pi_2 G_2$	$NR_{(t,k)}$
2	1	59.22	40	30	50	13.44	45.78	2961	1911	1050
2	2	45.66	30	30	30	0	45.66	1369.80	1369.80	0
1	1	52	30	30	40	0	52	2080	1560	520

By using Equation (2.11), $NPV_{(t,k)}$ (NPV of total profit gained in one year) for the states of the binomial lattice can be calculated in Table 2.3. We remark that the unit of $NPV_{(t,k)}$ is dollars per year.

Table 2.3 *NPV calculation - without investment*

t	k	$NR_{(t,k)}$	$NPV_{(t,k)}$
2	1	1050	8,509,714
2	2	0	-250,285
1	1	520	4,088,000

For the final lattice, for $t = 2$, we have to incur decommissioning cost by adding $(-250,000)(1 + 0.05)^{-1}$. Thus, for $t = 2$ and $k = 1$, NPV with decommissioning cost is \$8,271,619. For $t = 2$ and $k = 2$, NPV with decommissioning cost −\$488,381. For $t = 1$ and $k = 1$, in addition to the $NPV_{(t,k)}$ value in Table 2.3, we have to add the risk-neutral expected value of the successor states at the next time point. Thus,

$$4{,}088{,}000 + (0.66 \cdot 8{,}271{,}619 - 488{,}381 \cdot 0.34)(1 + 0.05)^{-1} = 9{,}123{,}428$$

Therefore, the lattice shown in Figure 2.8 without investment is obtained:

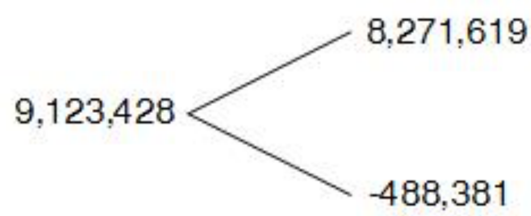

Figure 2.8 *The NPV lattice without investment ($)*

The NPV lattice - investment between centers 1 and 3

Option 1 (Investment at the beginning of the first period)

We assume that a power line is added between centers 1 and 3 at the beginning of the first period. We further assume that the capacity of the new line is 4 MW and it has the same susceptance with the existing power line. With this upgrade, fixed operation and maintenance cost increases to $40/h. The updated network can be seen in Figure 2.9. Because a new line is added to the network, underlying OPF problem formulation changes. It should be noted that the susceptance of the power line between centers 1 and 3 is now doubled (see Appendix 2.F for details).

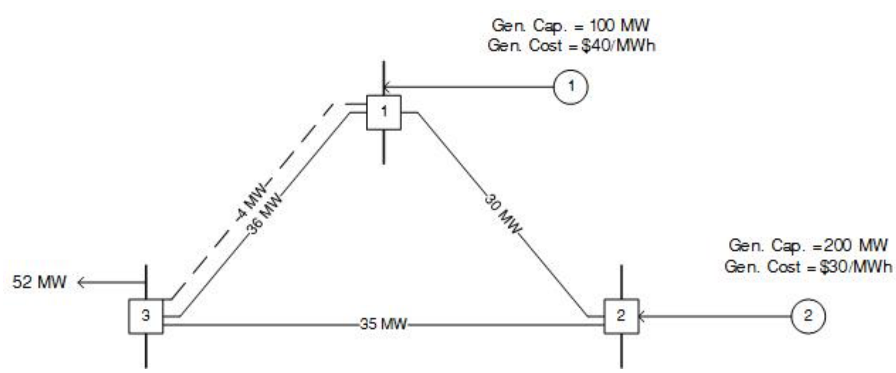

Figure 2.9 *Upgraded network - investment between centers 1 and 3*

The results of the LMP calculations for the upgraded network are given in Table 2.4 (see Appendix 2.G for details). The same remarks for Table 2.2 regarding the units of values are noted.

Table 2.4 *LMP calculation - investment between centers 1 and 3, Option 1*

t	k	$E_{(t,k)}$	π_1	π_2	π_3	G_1	G_2	$\pi_3 E_{(t,k)}$	$\pi_1 G_1 + \pi_2 G_2$	$NR_{(t,k)}$
2	1	59.22	40	30	45	1.33	57.89	2664.90	1789.90	875
2	2	45.66	30	30	30	0	45.66	1369.80	1369.80	0
1	1	52	30	30	30	0	52	1560	1560	0

By using Equation (2.11), $NPV_{(t,k)}$ for the states of the binomial lattice can be calculated as shown in Table 2.5.

Table 2.5 *NPV calculation - investment between centers 1 and 3, Option 1*

t	k	$NR_{(t,k)}$	$NPV_{(t,k)}$
2	1	875	6,966,285
2	2	0	-333,714
1	1	0	-333,714

We note that decommissioning cost of the network with a new power line is assumed to be \$300,000, which is larger than that of the network without investment.

For the final lattice, for $t = 2$, we have to incur decommissioning cost by adding $(-300,000)(1 + 0.05)^{-1}$. Thus, for $t = 2$ and $k = 1$, NPV with decommissioning cost is \$6,680,571. For $t = 2$ and $k = 2$, NPV with decommissioning cost is −\$619,429. For $t = 1$ and $k = 1$, in addition to $NPV_{(t,k)}$ value, we have to consider transmission access charge ($A = \$17M$) and initial investment cost ($I = \$15M$) as well as a risk-neutral expected value of the successor states at the next time point. Thus,

$$-333{,}714 + 17M - 15M + (0.66 \cdot 6{,}680{,}571 - 619{,}429 \cdot 0.34)(1 + 0.05)^{-1} = 5{,}660{,}147$$

Therefore, the lattice shown in Figure 2.10 with Option 1 can be obtained.

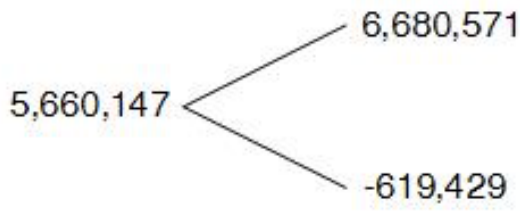

Figure 2.10 *The NPV lattice ($) - investment between centers 1 and 3, Option 1*

Because $5,660,147 is less than $9,123,428, the value of Option 1 is zero for this investment alternative.

Option 2 (investment at the beginning of the second period)

We assume that a power line is added between centers 1 and 3 at the beginning of the second year. Moreover, we assume that the capacity of this line is 4 MW and it has the same susceptance value with the existing line.

Table 2.6 gives the corresponding LMPs and LMP-based revenues. The same remarks for Table 2.2 regarding the units of values are noted. We note that with demand value equal to 52 MW, the LMP calculations are the same as those in the without investment situation because there is no investment at that time.

Table 2.6 *LMP calculation - investment between centers 1 and 3, Option 2*

t	k	$E_{(t,k)}$	π_1	π_2	π_3	G_1	G_2	$\pi_3 E_{(t,k)}$	$\pi_1 G_1 + \pi_2 G_2$	$NR_{(t,k)}$
2	1	59.22	40	30	45	1.33	57.89	2664.90	1789.90	875
2	2	45.66	30	30	30	0	45.66	1369.80	1369.80	0
1	1	52	30	30	40	0	52	2080	1560	520

By using Equation (2.11), $NPV_{(t,k)}$ for the states of the binomial lattice can be calculated as shown in Table 2.7 (but c is $40/h for the second period).

Table 2.7 *NPV calculation - investment between centers 1 and 3, Option 2*

t	k	$NR_{(t,k)}$	$NPV_{(t,k)}$
2	1	875	6,966,285
2	2	0	-333,714
1	1	520	4,088,000

For the final lattice, for $t = 2$, we have to incur decommissioning cost by adding $(-300,000)(1 + 0.05)^{-1}$. Moreover, A and I should be added and subtracted, respectively. Thus, for $t = 2$ and $k = 1$,

$$6{,}966{,}285 + 17M - 15M + (-300{,}000)(1 + 0.05)^{-1} = 8{,}680{,}571$$

For $t = 2$ and $k = 2$,

$$-333{,}714 + 17M - 15M + (-300{,}000)(1 + 0.05)^{-1} = 1{,}380{,}571$$

For $t = 1$ and $k = 1$, in addition to $NPV_{(t,k)}$ value, we have to add the risk-neutral expected value of the successor states at the next time point. Thus,

$$4{,}088{,}000 + (0.66 \cdot 8{,}680{,}285 + 0.34 \cdot 1{,}380{,}571)(1 + 0.05)^{-1} = 9{,}986{,}624$$

Therefore, the lattice shown in Figure 2.11 with Option 2 can be obtained.

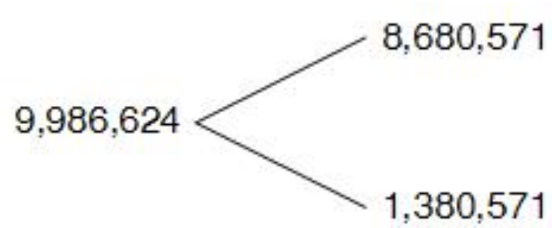

Figure 2.11 *The NPV lattice ($) - investment between centers 1 and 3, Option 2*

Because \$9,986,624 is larger than \$9,123,428 ($V_{(1,1)}$ value of the NPV lattice without investment), Option 2's value is found as the difference between these two values; that is, \$863,196. This is the value of investing at the beginning of the second period. By comparing Option 1 and Option 2, it is clear that Option 2 turns out to be valuable.

The NPV lattice - investment between centers 1 and 2

Option 1 (investment at the beginning of the first period)

We consider that an investment is made between centers 1 and 2 at the beginning of the first year. For consistency with the previous investment's parameters, the capacity of the new power line is assumed to be 4 MW and its susceptance is assumed to be equal to that of the current line. Similarly, we consider that operation and maintenance cost is again \$40/h. The upgraded network can be seen in Figure 2.12. Due to the change in the network configuration, a new OPF formulation should be devised because the susceptance value of the power line between centers 1 and 2 needs to be doubled (see Appendices 2.H and 2.I for the OPF problem and the LMP-based revenue derivations for the upgraded network).

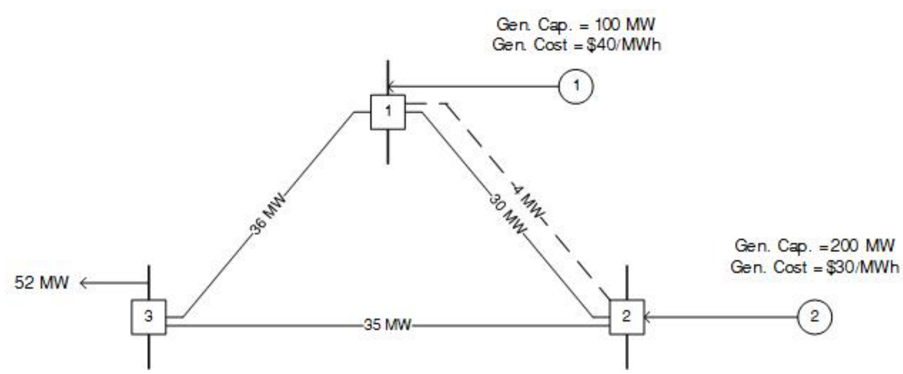

Figure 2.12 *Upgraded network - investment between centers 1 and 2*

The results of the LMP calculations for the upgraded network are given in Table 2.8. The same remarks for Table 2.2 regarding the units of values are noted.

Table 2.8 *LMP calculation - investment between centers 1 and 2, Option 1*

t	k	$E_{(t,k)}$	π_1	π_2	π_3	G_1	G_2	$\pi_3 E_{(t,k)}$	$\pi_1 G_1 + \pi_2 G_2$	$NR_{(t,k)}$
2	1	59.22	40	30	60	2.66	56.56	3553.20	1803.20	1750
2	2	45.66	30	30	30	0	45.66	1369.80	1369.80	0
1	1	52	30	30	30	0	52	1560	1560	0

By using Equation (2.11), $NPV_{(t,k)}$ for the states of the binomial lattice can be calculated as shown in Table 2.9.

Table 2.9 *NPV calculation - investment between centers 1 and 2, Option 1*

t	k	$NR_{(t,k)}$	$NPV_{(t,k)}$
2	1	1750	14,266,285
2	2	0	-333,714
1	1	0	-333,714

We note that decommissioning cost of the network is the same as in the previous investment. That is, we accept the same decommissioning cost, which is equal to \$300,000.

For the final lattice, for $t = 2$, we have to incur decommissioning cost by adding $(-300,000)(1 + 0.05)^{-1}$. Thus, for $t = 2$ and $k = 1$, NPV with decommissioning cost is \$13,980,571. For $t = 2$ and $k = 2$, NPV with decommissioning cost is −\$619,429. For $t = 1$ and $k = 1$, in addition to $NPV_{(t,k)}$ value, we have to consider transmission access charge ($A = \$17M$) and initial investment cost ($I = \$15M$) as well as a risk-neutral expected value of the successor states at the next time point. Thus,

$$-333{,}714 + 17M - 15M + (0.66 \cdot 13{,}980{,}571 - 619{,}429 \cdot 0.34)(1 + 0.05)^{-1} = 10{,}243{,}941$$

Therefore, the lattice shown in Figure 2.13 with Option 1 can be obtained:

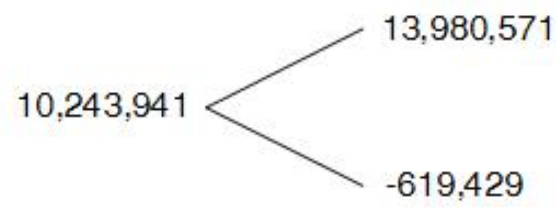

Figure 2.13 *The NPV lattice (\$) - investment between centers 1 and 2, Option 1*

Because \$10,243,941 is larger than \$9,123,428 ($V_{(1,1)}$ of the NPV lattice without investment), the value of Option 1 is found as \$1,120,513. This is the value of investing between centers 1 and 2 at the beginning of the first year.

Option 2 (investment at the beginning of the second period)

We assume that a power line is added between centers 1 and 2 at the beginning of the second year. Moreover, we assume that the capacity of this line is 4 MW and it has the same susceptance value with the existing line.

Table 2.10 gives the corresponding LMPs and LMP-based revenues. The same remarks for Table 2.2 regarding the units of values are noted. We note that with demand value equal to 52 MW, the LMP calculations are the same as those in the without investment situation because there is no investment at that time.

Table 2.10 *LMP calculation - investment between centers 1 and 2, Option 2*

t	k	$E_{(t,k)}$	π_1	π_2	π_3	G_1	G_2	$\pi_3 E_{(t,k)}$	$\pi_1 G_1 + \pi_2 G_2$	$NR_{(t,k)}$
2	1	59.22	40	30	60	2.66	56.56	3553.20	1803.20	1750
2	2	45.66	30	30	30	0	45.66	1369.80	1369.80	0
1	1	52	30	30	40	0	52	2080	1560	520

By using Equation (2.11), $NPV_{(t,k)}$ for the states of the binomial lattice can be calculated as shown in Table 2.11 (but c is \$40/h for the second period).

Table 2.11 *NPV calculation - investment between centers 1 and 2, Option 2*

t	k	$NR_{(t,k)}$	$NPV_{(t,k)}$
2	1	1750	14,266,285
2	2	0	-333,714
1	1	520	4,088,000

For the final lattice, for $t = 2$, we have to incur decommissioning cost by adding $(-300{,}000)(1 + 0.05)^{-1}$. Moreover, A and I should be added and subtracted, respectively. Thus, for $t = 2$ and $k = 1$,

$$14{,}266{,}285 + 17M - 15M + (-300{,}000)(1 + 0.05)^{-1} = 15{,}980{,}571$$

For $t = 2$ and $k = 2$,

$$-333{,}714 + 17M - 15M + (-300{,}000)(1 + 0.05)^{-1} = 1{,}380{,}571$$

For $t = 1$ and $k = 1$, in addition to $NPV_{(t,k)}$ value, we have to add risk-neutral expected value of the successor states at the next time point. Thus,

$$4{,}088{,}000 + (0.66 \cdot 15{,}980{,}571 + 0.34 \cdot 1{,}380{,}571)(1 + 0.05)^{-1} = 14{,}570{,}417$$

Therefore, the lattice shown in Figure 2.14 with Option 2 is obtained:

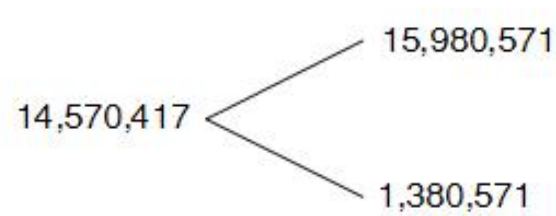

Figure 2.14 *The NPV lattice ($) - investment between centers 1 and 2, Option 2*

Because \$14,570,417 is larger than \$9,123,428 ($V_{(1,1)}$ of the NPV lattice without investment), the value of Option 2 is found as \$5,446,990. Thus, it can be said that the value of investing between centers 1 and 2 at the beginning of the second year is \$5,446,990. Because the value of Option 2 is larger than that of Option 1, Option 2 becomes more likely to be implemented.

The NPV lattice - investment between centers 2 and 3

Option 1 (investment at the beginning of the first period)

We consider that another power line is added between centers 2 and 3. To maintain consistency with the parameters of previous investment alternatives, the capacity of the new line is assumed to be 4 MW and susceptance of it is equal to that of the existing power line between centers 2 and 3. The upgraded network can be seen in Figure 2.15. In order to reformulate the OPF problem for the upgraded network, susceptance value of the power line between centers 2 and 3 should be doubled (see Appendices 2.J and 2.K for the OPF problem and the LMP-based revenue derivations for the upgraded network).

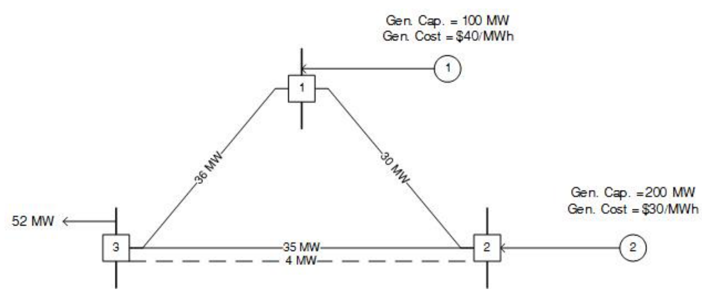

Figure 2.15 *Upgraded network - investment between centers 2 and 3*

For the network with the corresponding new line, the results of the LMP calculations are given in Table 2.12. The same remarks for Table 2.2 regarding the units of values are noted.

Table 2.12 *LMP calculation - investment between centers 2 and 3, Option 1*

t	k	$E_{(t,k)}$	π_1	π_2	π_3	G_1	G_2	$\pi_3 E_{(t,k)}$	$\pi_1 G_1 + \pi_2 G_2$	$NR_{(t,k)}$
2	1	59.22	40	30	50	20.94	38.28	2961	1986	975
2	2	45.66	30	30	30	0	45.66	1369.80	1369.80	0
1	1	52	40	30	50	6.5	45.5	2600	1625	975

By using Equation (2.11), $NPV_{(t,k)}$ for the states of the binomial lattice can be calculated as shown in Table 2.13.

Table 2.13 *NPV calculation - investment between centers 2 and 3, Option 1*

t	k	$NR_{(t,k)}$	$NPV_{(t,k)}$
2	1	975	7,800,571
2	2	0	-333,714
1	1	975	7,800,571

For the final lattice, for $t = 2$, we have to incur decommissioning cost by adding $(-300,000)(1 + 0.05)^{-1}$. Thus, for $t = 2$ and $k = 1$, NPV with decommissioning cost is \$7,514,857. For $t = 2$ and $k = 2$, NPV with decommissioning cost is $-$\$619,429. For $t = 1$ and $k = 1$, in addition to $NPV_{(t,k)}$ value, we have to consider transmission access charge ($A = \$17M$) and initial investment cost ($I = \$15M$) as well as a risk-neutral expected value of the successor states at the next time point. Thus,

$$7{,}800{,}571 + 17M - 15M + (0.66 \cdot 7{,}514{,}857 - 619{,}429 \cdot 0.34)(1 + 0.05)^{-1} = 14{,}318{,}295$$

Therefore, the lattice shown in Figure 2.16 with Option 1 can be obtained:

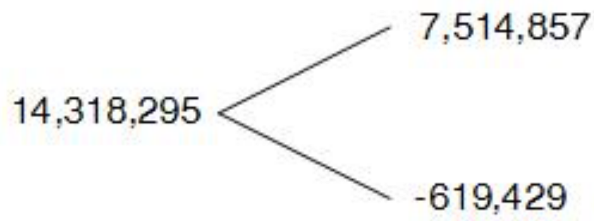

Figure 2.16 *The NPV lattice ($) - investment between centers 2 and 3, Option 1*

Because $14,318,295 is larger than $9,123,428 ($V_{(1,1)}$ of the NPV lattice without investment), Option 1's value is found as $5,194,868. This is the value of investing between centers 2 and 3 at the beginning of the first year.

Option 2 (investment at the beginning of the second period)

We assume that a power line is added between centers 2 and 3 at the beginning of the second year. Moreover, we assume that the capacity of this line is 4 MW and it has the same susceptance value with the existing line.

Table 2.14 gives the corresponding LMPs and LMP-based revenues. The same remarks for Table 2.2 regarding the units of values are noted. We note that with demand value being equal to 52 MW, the LMP calculations are the same as those in the without investment situation because there is no investment at that time.

Table 2.14 *LMP calculation - investment between centers 2 and 3, Option 2*

t	k	$E_{(t,k)}$	π_1	π_2	π_3	G_1	G_2	$\pi_3 E_{(t,k)}$	$\pi_1 G_1 + \pi_2 G_2$	$NR_{(t,k)}$
2	1	59.22	40	30	50	20.94	38.28	2961	1986	975
2	2	45.66	30	30	30	0	45.66	1369.80	1369.80	0
1	1	52	30	30	40	0	52	2080	1560	520

By using Equation (2.11), $NPV_{(t,k)}$ for the states of the binomial lattice can be calculated as shown in Table 2.15 (but c is $40/h for the second period).

Table 2.15 *NPV calculation - investment between centers 2 and 3, Option 2*

t	k	$NR_{(t,k)}$	$NPV_{(t,k)}$
2	1	975	7,800,571
2	2	0	-333,714
1	1	520	4,088,000

For the final lattice, for $t = 2$, we have to incur decommissioning cost by adding $(-300{,}000)(1 + 0.05)^{-1}$. Moreover, A and I should be added and subtracted, respectively. Thus, for $t = 2$ and $k = 1$,

$$7{,}800{,}571 + 17M - 15M + (-300{,}000)(1 + 0.05)^{-1} = 9{,}514{,}857$$

For $t = 2$ and $k = 2$,

$$-333{,}714 + 17M - 15M + (-300{,}000)(1 + 0.05)^{-1} = 1{,}380{,}571$$

For $t = 1$ and $k = 1$, in addition to $NPV_{(t,k)}$ value, we have to add the risk-neutral expected value of the successor states at the next time point. Thus,

$$4{,}088{,}000 + (0.66 \cdot 9{,}514{,}857 + 0.34 \cdot 1{,}380{,}571)(1 + 0.05)^{-1} = 10{,}510{,}486$$

Therefore, the lattice shown in Figure 2.17 with Option 2 is generated:

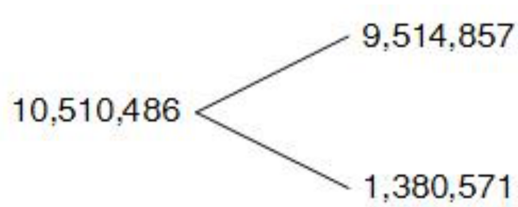

Figure 2.17 *The NPV lattice (\$) - investment between centers 2 and 3, Option 2*

Because \$10,510,486 is larger than \$9,123,428 ($V_{(1,1)}$ of the NPV lattice without investment), the value of Option 2 is found as \$1,387,058. This is the value of investing between centers 2 and 3 at the beginning of the second year. Of Option 1 and Option 2, the first one is more preferable because it has larger value.

After all of these calculations, we can present Table 2.16 as a summary for all investment alternatives.

Table 2.16 *Investment alternatives, their values and times*

Investment Alternatives	Values	Timing of the Investments
Centers 1 - 3	\$863,196	2
Centers 1 - 2	\$5,446,990	2
Centers 2 - 3	\$5,194,868	1

The owner has two different flexibilities. One is that he or she can expand the network or not because expansion is not an obligatory issue. The other flexibility is that if the owner decides to invest, he or she can defer the investment, which means that he or she can invest at the beginning of any year. We clarify that these flexibilities cannot be exercised independently.

As can be seen, Table 2.16 shows only the investment values for each investment alternative at the time when it is optimal to invest. In other words, for an investment alternative, the second column indicates the investment value that is the maximum of the values of Option 1 (making the investment at the beginning of the first year) and Option 2 (making the investment at the beginning of the second year). The third column shows the corresponding time in which the maximum occurs. For example, for the investment alternative between centers 1 and 2, the computational results reveal that Option 1 and Option 2 have values of \$1,120,513 and \$5,446,990, respectively. Thus, the value of the investment at the beginning of the second year is given as the maximum in Table 2.16 and the corresponding time is determined as the optimal time of the investment.

For the investment alternative between centers 1 and 3, Table 2.16 shows that the value of Option 2 is \$863,196. With regard to the value of Option 1, it is calculated as \$0 in the preceding sections and it is not shown in Table 2.16 because it turns out to be less than Option 2's value. It is inferred from these results that the decision maker expands the network by installing a transmission line between centers 1 and 3 and this investment is deferred to the beginning of the second year. We note that the decision maker is assumed to behave optimally.

As for the investment alternative between centers 1 and 2, Table 2.16 presents the value of Option 2 as \$5,446,990. The value of Option 1, not shown in Table 2.16, is calculated as \$1,120,513. Therefore, it can be inferred that the decision maker decides to expand the network by investing in a transmission line between centers 1 and 2, and it is carried out after postponing it for one year.

The investment alternative between centers 2 and 3 results in different outcomes. More specifically, Table 2.16 demonstrates the value of Option 1 because it turns out to be larger than the value of Option 2. The value of Option 1 is determined as \$5,194,868. However, it is revealed that Option 2 has a value of \$1,387,058. This implies that the decision maker decides to expand the network by adding a transmission line between centers 2 and 3, but this investment is not deferred.

As stated above, the investment made between centers 2 and 3 behaves differently relative to the investments between centers 1 and 2 as well as between centers 1 and 3 as follows. Specifically, investing at the beginning of the first year is the most preferable because more revenues are gained. The reason is that because an added power line has the same susceptance (thus total susceptance is doubled on that circuit), it dramatically changes the network configuration and more power tries to flow on that circuit. However, because the capacity of the new line is very low (4 MW) with respect to the capacity of the existing power line (35 MW), this increases congestion and increases the revenue due to the increase in differences between the LMPs. Therefore, the investor is not in favor of deferring this investment.

As for the investment made between centers 1 and 2, investing at the beginning of the second year is more preferable because more revenue is generated throughout the first year if no investment is made at the beginning of the first year. The reason is that a new power line changes the network configuration, but it decreases congestion and decreases revenue due to the decrease in differences between the LMPs. Therefore, the current set of parameters is in favor of delaying the investment and the investor tends to defer it to get more revenue throughout the first year.

For the investment made between centers 1 and 3, making an investment in this circuit at the beginning of the second year is also more preferable because more revenue is generated throughout the first year if any investment is not made at the beginning of the first year. The reason is that a newly added power line changes the network configuration in favor of decreasing congestion and decreasing revenue generated by differences between the LMPs. Thus, the current set of parameters is in favor of postponing the investment and the investor defers it to gain more revenue throughout the first year.

Further Discussions

A critical question might arise related to whether stochastic processes different from GBM can be incorporated into the developed framework. We can state that there are several attempts in the literature to approximate other stochastic processes by lattice approaches. These studies can be classified into two groups: one that seeks to develop the binomial lattices and another that puts effort to construct the trinomial lattices.

In the first group, Nelson and Ramaswamy (1990) presented a method to develop recombining binomial lattices for the stochastic processes other than GBM such as Cox-Ingersoll-

Ross (Cox et al. 1985) and Ornstein-Uhlenbeck processes. In another study, Bastian-Pinto et al. (2010) developed a recombining binomial lattice for mean-reverting processes by matching the expected value and the variance of the underlying continuous and its discrete counterpart processes. The lattice model of Nelson and Ramaswamy (1990) was extended by Hahn and Dyer (2008) to the discretization of two correlated Ornstein-Uhlenbeck processes. The extended model is employed in order to evaluate the real options. Slade (2001) made use of the binomial lattices developed by Nelson and Ramaswamy (1990) to model the mean-reverting copper price and unit cost evolutions to evaluate the managerial flexibilities in mining operations. There are many other studies that use the binomial lattice models to discretize the stochastic processes that are different from GBM (see, e.g., Bastian-Pinto et al. 2009; Lari-Lavassani et al. 2001).

In the second group, the researchers aim to find the trinomial lattices to approximate the stochastic processes different from GBM. For instance, Jaillet et al. (2004) took advantage of a trinomial lattice to model the underlying uncertainty that follows the Ornstein-Uhlenbeck process. Yet another study conducted by Tseng and Lin (2007) pursued the development of a bivariate trinomial lattice for two correlated mean-reverting processes.

Another crucial question might arise as to why a short term is selected as the modeling horizon in the numerical example. The reason is that our fundamental goal is to derive the managerial insights from the framework by keeping the model as simple as possible. If the modeling horizon were expanded to many years, it would be challenging to find the policy insights. As a matter of fact, there exist many core studies in the literature in line with this consideration. Those studies likewise employ the small-scale dynamic programming or lattice models.

For instance, Dixit and Pindyck (1994), which is probably the most remarkable reference in real options literature, expressed their ideas as to the value of the options with a small-scale dynamic programming model. According to this model, the current price of an item will increase or decrease by a constant amount at the end of the first year and then the new price will stay at the same level forever. The authors explained their intents in relation to why the matter is kept so simple as follows: “It is best to begin with some simple examples, involving a minimal amount of mathematics, in which investment decisions are made at two or three discrete points in time. In this way, we can convey at the outset an intuitive understanding of the basic concept” (p. 26). Similarly, Luenberger (1997) conveyed the core ideas regarding dynamic pricing by presenting straightforward dynamic programming models that commonly involve a few periods. For instance, a fishing example (Luenberger 1997, p. 117) has three periods and a gold mine example (Luenberger 1997, p. 347) has 10 periods. The main purpose of the author is to make the central ideas understandable.

As for the transmission investment literature, in the study by Blanco et al. (2009), two different real options in transmission investments are evaluated. These options have maturity equal to 2 years and, therefore, the binomial lattices have just two 1-year periods. In a related study, Blanco et al. (2012) exploited the stochastic dynamic programming approach to evaluate the value of flexibility in transmission investments. In the numerical example, the authors made the assumption that building permits to install the transmission lines are valid for 3 years, which results in a 3-year dynamic programming model. In transmission investment literature, several other research works can be found that prefer using small-scale dynamic programming or lattice models (see, e.g., Loureiro et al. 2015; Vásquez and Olsina 2010).

The use of small-scale dynamic programming models can be found in other application areas as well, such as stockpiling of oil. For example, Bai et al. (2012) sought an optimal stockpiling path for China’s petroleum reserve between the years 2008 and 2020. The authors built and solved a 12-year dynamic programming model to reach the goal. A similar study was carried out by Wu et al. (2008), who were concerned with finding an optimal stockpile acquisition strategy for China in the time intervals 2007–2010 and 2011–2020. A 4-year dynamic programming model was built for the first time interval. Furthermore, Wu et al. (2012) dealt with China's optimal stockpiling and drawdown strategies for petroleum reserves. The authors pointed out that this problem is dynamic in nature because the country has to determine the level of acquisition and release in each year. For this purpose, a dynamic programming model spanning 10 years was introduced and implemented.

If a much longer modeling horizon was selected such as 40 years, the quality of the solution would suffer from the quality of old data/input/parameter values. For instance, what was projected in a 1976 study as an outcome of 2016 would certainly be different from the actual observations in 2016 not only in terms of numerical value but also in terms of the nature and context of the changed business environment.

We again note that we prefer to study a small-scale lattice model in the numerical example because our essential purpose is to come up with some managerial insights while keeping the problem size as small as possible. For researchers who follow a similar strategy in their models, readers can refer to the studies mentioned above.

As our framework indicates, we consider the decommissioning cost as the terminal condition. Keeping this in view, one question might arise as to why the residual values of the transmission assets are not considered. In general, the cost of decommissioning can be negative, which implies that residual value might be significant. In our model, the cost of decommissioning is a parameter but not a central parameter. We leave the incorporation of residual value into the model as a future study.

Concluding Remarks and Future Research

In this article, we developed and analyzed a real options framework that provides the valuation of a transmission owner's option to expand in his or her network. Specifically, under the assumption that the evolution of the demand follows a GBM process, our framework explicitly accounts for the physical flow of the electric power - economically manifested as the LMPs. Through this framework, we show how the values of the expansion options can be determined in the transmission network. Moreover, given that a specific expansion is already planned, we show how to value an option to expedite or delay. An extensive numerical example is provided to illustrate the key features of our framework with interesting managerial insights.

We note that the framework in this article can be used as a basis for several expanded studies. For example, additional uncertainties such as fuel costs and regulatory changes can be incorporated. At this point, some questions might arise as to the modeling of fuel prices and changes in regulatory framework. A large number of studies in the literature support the idea that GBM can be employed to model fuel prices. For instance, Postali and Picchetti (2006) devoted a whole paper to discussing the appropriateness of GBM to represent the evolution of fuel prices. An interesting finding from the empirical tests is that the reversion speed in mean-reverting process is too low. Therefore, GBM can be utilized as a good proxy to the evolution of oil prices. The overall results of empirical tests led the authors to reach the conclusion that using GBM does not lead to a significant error in real options evaluations. Because that is the case, it can be adopted by research practitioners due to its advantage of obtaining analytical solutions. In another study, Gibson and Schwartz (1990) tested the hypothesis that the spot price of oil is lognormally distributed. Having collected weekly data between 1984 and 1988, the authors observed that the spot price of oil presents a random walk behavior and historical volatility appears to be stable across the periods. It shows that GBM can be used to model the oil price evolution. There are also other studies in the literature that model fuel price evolutions with GBM (see, e.g., Aronne et al. 2008).

As for the changes in regulatory framework, GBM is not an appropriate process to model this because there cannot be a change in regulatory framework in each tiny time interval, which should be the case in GBM. However, jump processes are generally utilized to model the evolution of the changes in regulatory framework. For example, Hassett and Metcalf (1999) investigated how tax policy uncertainty affects the investment decision of firms. Having focused on a hypothetical firm, the authors modeled the after-tax price of the product with GBM. Because tax policy changes affect this price in a discrete manner, its evolution is modeled with a Poisson jump process embedded in a GBM process.

As for the discrete versions of Poisson jump along with GBM processes, the literature has noteworthy studies that combine two processes in a single lattice model. For example, Amin (1993) made one of the first attempts in discretizing Poisson jump-GBM processes. In his lattice model, whereas a movement to one state above or below in the next time point represents GBM process, the movement to more than one state above or below mimics the jump process. It is important to emphasize that Amin (1993) discretized GBM and the jump processes in the same grid, which means that states in the vertical space are located equidistantly and each state represents either a jump or GBM event. Yet another prominent study carried out by Hilliard and Schwartz (2005) distinguishes itself from Amin (1993) by analyzing GBM and Poisson jump processes on separate grids. In other words, the distance between jump states and the distance between GBM states are formulated differently. Additionally, the way of calculating the jump branch probabilities is distinct from that of Amin (1993). In a different study, Martzoukos and Trigeorgis (2002) extended the model of Amin (1993) to multiple types of Poisson jumps, which means that there are multiple sources of events to induce the diffusion process to make a jump. Though this model is structurally the same as in Amin (1993), one fundamental difference can be mentioned that a jump event is assumed to happen after a GBM event occurs in a tiny time interval. More studies can be found in the literature that discretize Poisson jump-GBM processes with a lattice approach (see, e.g., Dai et al. 2010).

Therefore, the above discussions related to fuel prices and changes in regulatory frameworks indicate that both uncertain factors can be incorporated into the developed lattice framework. To put it briefly, the evolution of fuel prices can be modeled with GBM and changes in regulatory framework can be represented in discrete Poisson jump processes. We note that if it is desired to embed the change in regulatory framework into the developed model, the current lattice model should be extended to the lattice framework of Poisson jump-GBM processes.

Another extension of the current study would be that more computationally intense models can be considered where the number of periods extends into the hundreds (e.g., a 10-year span with a potential decision point in each month). Through such realistic extensions, we hope that this line of study will be helpful in understanding the critical issues in transmission expansion planning faced with substantial and increasing uncertainties in the near future.

Yet another extension of the current model would be to consider both the residual value and the decommissioning cost in the terminal states of the lattice model. This will hopefully facilitate observing the effect of the residual value on the investment decisions.

Appendix 2.A OPF Problem

The OPF problem is a power flow configuration to operate an electrical system in a best way. It is an optimization problem that results in the best way to operate the system. For an AC network, the decision variables of the OPF problem are the voltage magnitudes and angles in the centers, power flow amounts on the power lines, and amounts of power dispatched from the generation centers.

For transmission networks, it is sufficient and legitimate to use the linearized form of the AC OPF formulation. The resulting form of the OPF is called direct current (DC) OPF and the decision variables can be listed as voltage angles in the centers, power flow amounts on the power lines, and amounts of power dispatched from the generation centers.

In DC OPF, the objective function generally arises as the minimization of the total cost of power generations in the network. As for the constraints, nodal balance requirements (the amount of power entering into a center should be equal to the amount of power emanating from this center) are placed for each center. In addition, power line capacities and capacities of power generation centers should not be exceeded. We note that Kirchhoff current and voltage laws are represented in DC OPF formulations. Throughout this study, we use a DC OPF formulation.

In electricity market, generators and consumers offer their hourly bid by considering their marginal cost or benefit functions. In reality, for strategic purposes, it is possible that suppliers and consumers do not bid their real marginal cost or benefit functions. However, in network planning, it is legitimate to assume that system operator can guess their average behavior that represents their real cost or benefit functions.

It can be stated that generator's marginal cost can be estimated because of their strategic behavior. In other words, a producer of electricity most likely offers bid at its marginal cost. The reason is that if the producer bid higher than its marginal cost, then the producer could be extra-marginal producer, which means he/she could not sell any energy. Moreover, if the producer bid lower than its marginal cost, then he/she could produce at a loss. Finally, if he/she bid at his marginal cost, then the producer would be paid at market clearing price and would make profit in the case that he/she is marginal or infra-marginal producer. Therefore, it can be concluded that in order the producer to make money, there is no other incentive other than bidding at the marginal cost. Even in this case, he/she should expect to be infra-marginal or marginal producer to make money.

Since it is assumed that system operator can estimate generator's marginal cost by observing the bidding strategy, we think that the OPF problem can be solved with the offered bid (or, marginal cost by the assumption that they are equal). In this chapter, we adopt that the OPF problem is solved with the marginal cost of generators that can be estimated by bidding strategy.

In fact, generators are dispatched in order to maximize the social welfare, which is defined as difference between total benefit obtained by the customers and total cost incurred by the generators. If demand is price-insensitive, then the problem turns into the minimization of generation cost. In this case, generators are dispatched with the least cost. In this study, we assume that demand is price-insensitive.

Mathematical presentation of the OPF problem is given below. In addition to notations used in the main text, we define the following notations:

- N: The set of all centers in the network
- M: The set of power lines in the network

- $\mathcal{B}_{ij}$: An element in the susceptance matrix (Siemens)
- b_{ij}: The susceptance of the power line between centers i and j (Siemens)
- θ_i: The voltage angle at center i (Radian)
- L_{ij}: The amount of power flows between centers i and j (MW)

The OPF problem is given as Equations (2A.1) - (2A.6). Objective function minimizes total generation cost in the network. The constraints are AC approximated by DC power flow equations (For approximation details, see Appendix 2.C).

Equation (2A.2) represents the power balance expression for each center. In fact, this balance can be called Kirchhoff Current Law, which states that amount of power entering into a center is equal to the amount of power emanating from this center. If there does not exist any generator at center i, then G_i becomes zero. Similarly, if there is not a consumption center at center i, then D_i is equal to zero.

Equation (2A.3) expands $\mathcal{B}_{ij}$ known as the element at i^{th} row and j^{th} column in the susceptance matrix. Susceptance matrix is a significant network analysis tool in power systems. Its significance originates from the fact that a computer program can solve the OPF problem of a huge power network by using the susceptance matrix as an input. As can be seen in Equation (2A.3), $\mathcal{B}_{ij}$ is just an element in susceptance matrix consisting of actual susceptance values b_{ij} of the power lines. Susceptance of a power line is defined as the measure of how easily electrical current flows on this line. We note that if there is not any power line between centers i and j, then corresponding b_{ij} value becomes zero.

Equation (2A.4) calculates the power amount flowing on the line connecting centers i and j. This equation is called as Kirchhoff Voltage Law because it is said to implicitly take into account the law due to the fact that it is an expression for Ohm's law. This is justified in a way that Equation (2A.4) includes the potential function θ_i and θ_j.

Constraints (2A.5) and (2A.6) present the thermal limit constraints of the power lines and production capacity constraints for the generators. If there does not exist any power line between centers i and j, then $\overline{L_{ij}}$ is accepted as zero.

$$\min \sum_{i \in N_G} C_i G_i \tag{2A.1}$$

$$\text{subject to} \quad G_i - D_i = \sum_{j=1}^{|N|} \mathcal{B}_{ij}\theta_j, \quad \forall i \in N \tag{2A.2}$$

$$\mathcal{B}_{ij} = \begin{cases} -b_{ij}, & if\ i \neq j \\ \displaystyle\sum_{j=1, j\neq i}^{|N|} b_{ij}, & if\ i = j \end{cases} \quad \forall i,j \in N \tag{2A.3}$$

$$L_{ij} = b_{ij}(\theta_i - \theta_j), \quad \forall i,j \in N \tag{2A.4}$$

$$-\overline{L_{ij}} \leq L_{ij} \leq \overline{L_{ij}}, \quad \forall (i,j) \in M \tag{2A.5}$$

$$0 \leq G_i \leq \overline{G_i}, \quad \forall i \in N_G \tag{2A.6}$$

For details on the OPF problem, please see Kirschen and Strbac (2004) and McCalley (2007).

Appendix 2.B Derivation of LMP Differences

Given a network of transmission power lines, the price of the electricity is determined at specific locations and it is called as the LMP. The LMP consists of the generation cost, line losses and network constraints. It is significant for electricity market because it represents the market clearing price of energy.

The LMP at center i can be calculated as follows: Firstly, the OPF problem is solved with the given demand values. Then demand value at center i is increased by 1 MW and the OPF problem is solved again. The difference between objective function values gives the LMP at center i.

Appendix 2.C Approximation of AC Power Flow Equations

Instead of dealing with AC OPF problem, one can linearize the AC power flow equations and reformulate the problem as DC OPF. For high-voltage transmission lines, certain real-life observations facilitate the derivation of DC power flow equations. In summary, these observations can be listed as follows:

1. The resistance of the power lines is extremely less than the reactance (which leads to the elimination of conductance from the AC power flow equations).
2. The difference in voltage angles of two centers is around 10–15° (which leads to relaxation regarding the cosine and sine functions in AC power flow equations).
3. In a per unit system, the voltage magnitudes at centers are close to one (which leads to the elimination of voltage magnitudes from the AC power flow equations).

Before elaborating, note that admittance of a power line is mathematically defined as $y = g + jb$ where g is the conductance and b is the susceptance of the power line. We note that j here denotes the imaginary unit of admittance, not the index of a center in the network. Furthermore, impedance of a power line is mathematically defined as $z = r + jx$ where r and x are resistance and reactance of the power line. It is well known that admittance y is just the reciprocal of impedance z.

AC power flows equations are written as follows:

$$P_k = \sum_{i=1}^{|N|} |\mathcal{V}_k||\mathcal{V}_i| \left(W_{ki} \cos \theta_k - \theta_i + \mathcal{B}_{ki} \sin \theta_k - \theta_i\right) \tag{2C.1}$$

$$Q_k = \sum_{i=1}^{|N|} |\mathcal{V}_k||\mathcal{V}_i| \left(W_{ki} \sin \theta_k - \theta_i - \mathcal{B}_{ki} \cos \theta_k - \theta_i\right) \tag{2C.2}$$

where P_k and Q_k denotes the net injected real and reactive power at center k. Net injected power can be found by subtracting demanded power from injected power at center k. Moreover, $|\mathcal{V}_k|$ and $|\mathcal{V}_i|$ denotes the voltage magnitude at centers k and i, respectively. W_{ki} and $\mathcal{B}_{ki}$ are the elements in conductance and susceptance matrices.

As stated in the main text, $\mathcal{B}_{ki}$ can be expanded as

$$\mathcal{B}_{ki} = \begin{cases} -b_{ki}, & if\ k \neq i \\ \sum_{i=1, k\neq i}^{|N|} b_{ki}\,, & if\ k = i \end{cases} \tag{2C.3}$$

where b_{ki} is the susceptance of power line connecting centers k and i. Similarly, W_{ki} can be expanded as

$$W_{ki} = \begin{cases} -g_{ki}, & if\ k \neq i \\ \sum_{i=1,k\neq i}^{|N|} g_{ki}\,, & if\ k = i \end{cases} \tag{2C.4}$$

where g_{ki} is the conductance of power line connecting centers k and i.

Observation 1: It is said that reactance of transmission lines is importantly greater than their resistance. Thus, admittance y can be calculated by the serial equations

$$y = \frac{1}{z} = \frac{1}{r + jx} \times \frac{r - jx}{r - jx} = \frac{r}{r^2 + x^2} - \frac{jx}{r^2 + x^2} = g + jb \tag{2C.5}$$

Since x is very large when compared to r, b will be very large compared to g. Thus, it is appropriate to approximate g and b as $g = 0$ and $b = \frac{-1}{x}$. Therefore, it can be assumed $W_{ki} = 0$. After this observation, power flow equations turn into the following form:

$$P_k = \sum_{i=1}^{|N|} |\mathcal{V}_k||\mathcal{V}_i|\ (\mathcal{B}_{ki}\, sin\, \theta_k - \theta_i) \tag{2C.6}$$

$$Q_k = \sum_{i=1}^{|N|} |\mathcal{V}_k||\mathcal{V}_i|\ (-\mathcal{B}_{ki}\, cos\, \theta_k - \theta_i) \tag{2C.7}$$

Observation 2: For almost all operating conditions, the difference $\theta_k - \theta_i$ is less than 10 - 15 degrees. If we consider cosine and sine function of such a small angle, we can reach a simplification. It is known that if angle goes to 0, then cosine of this angle goes to 1 and its sine goes to angle itself. If we apply these relationships on the equations, we get

$$P_k = \sum_{i=1}^{|N|} |\mathcal{V}_k||\mathcal{V}_i|\ (\mathcal{B}_{ki}(\theta_k - \theta_i)) \tag{2C.8}$$

$$Q_k = \sum_{i=1}^{|N|} |\mathcal{V}_k||\mathcal{V}_i|\ (-\mathcal{B}_{ki}) \tag{2C.9}$$

Equation (2C.9) can be written by separating it into parts, which includes $\mathcal{B}_{kk}$ and $\mathcal{B}_{kj}$. Thus,

$$Q_k = \sum_{i=1}^{|N|} |\mathcal{V}_k||\mathcal{V}_i|\ (-\mathcal{B}_{ki}) = -|\mathcal{V}_k|^2 \mathcal{B}_{kk} - \sum_{i=1,i\neq k}^{|N|} |\mathcal{V}_k||\mathcal{V}_i|\ (\mathcal{B}_{ki}) \tag{2C.10}$$

We can expand $\mathcal{B}_{kk}$ and $\mathcal{B}_{ki}$ by using Equation (2C.3). After doing algebraic operations, we get

$$Q_k = -|\mathcal{V}_k|^2 b_k - \left(\sum_{i=1, i\neq k}^{|N|} |\mathcal{V}_k| b_{ki} (|\mathcal{V}_k| - |\mathcal{V}_i|) \right) \tag{2C.11}$$

The second term of Equation (2C.11) is reactive power flowing on the line connecting the centers k and i. It is proportional to voltage magnitude at center k and voltage magnitude differences of centers k and i.

As for real power flow equation given in Equation (2C.8), we can also separate it into two parts with $\mathcal{B}_{kk}$ and $\mathcal{B}_{ki}$. Thus,

$$P_k = |\mathcal{V}_k|^2 \big(\mathcal{B}_{kk}(\theta_k - \theta_k)\big) + \sum_{i=1, i\neq k}^{|N|} |\mathcal{V}_k||\mathcal{V}_i| \left(\mathcal{B}_{ki}(\theta_k - \theta_i)\right) \tag{2C.12}$$

and

$$P_k = \sum_{i=1, i\neq k}^{|N|} |\mathcal{V}_k||\mathcal{V}_i| \left(\mathcal{B}_{ki}(\theta_k - \theta_i)\right) \tag{2C.13}$$

From Equation (2C.13), we can say that real power flowing on the power line connecting centers k and i is proportional to voltage magnitudes at centers k and i and voltage angle difference at those centers.

Observation 3: The voltage magnitudes $|\mathcal{V}_k|$ and $|\mathcal{V}_i|$ are very close 1 in per-unit system. Usual range for the voltage magnitudes under most conditions are $[0.95, 1.05]$. Thus, if we assume $|\mathcal{V}_k| = |\mathcal{V}_i| = 1$, then we make a small and negligible mistake in the multiplication term $|\mathcal{V}_k||\mathcal{V}_i|$. On the other hand, by the same assumption, we make a huge mistake in the difference term $|\mathcal{V}_k| - |\mathcal{V}_i|$. For example, let's consider the worst case, which is the case that $|\mathcal{V}_k| = 1.05$ and $|\mathcal{V}_i| = 0.95$. In that case, the multiplication result is 0.9975 and the difference term equals to 0.1. If we assume that the voltage magnitude is equal to 1, then multiplication result in 1 and difference equals to 0. Hence, the mistake in multiplication is $1 - 0.9975 = 0.0025$ and the mistake in difference is $0.1 - 0 = 0.1$. After comparing two mistakes, we can say that it can be assumed the voltage magnitudes are equal to 1 in real power flow equation. However, we cannot make the same assumption for reactive power flow equation, except for inserting 1 in the place of single $|V_k|$. Therefore, the last form of power flow equations is

$$P_k = \sum_{i=1, i\neq k}^{|N|} \mathcal{B}_{ki}(\theta_k - \theta_i) \tag{2C.14}$$

$$Q_k = -b_k - \left(\sum_{i=1, i \neq k}^{|N|} b_{ki} (|\mathcal{V}_k| - |\mathcal{V}_i|) \right) \tag{2C.15}$$

We observe that reactive power flow is proportional to circuit susceptance and voltage magnitude differences. The maximum difference between voltage magnitude is $1.05 - 0.95 = 0.1$. On the other hand, the real power flow is proportional to circuit susceptance and voltage angle differences. The maximum difference between voltage angles are 0.52 radian, which equals to 30°. Thus, real power flow is significantly greater than reactive power flow. Finally, with these observations, we state that in approximated power flow equations, it is sufficient to consider only real power flows. Thus, AC approximated by DC power flow equations is given as

$$P_k = \sum_{i=1, i \neq k}^{|N|} \mathcal{B}_{ki} (\theta_k - \theta_i) \tag{2C.16}$$

For more detail regarding the approximation procedure, please see the relevant references Kirschen and Strbac (2004) and McCalley (2007).

Appendix 2.D OPF (Without Investment)

As stated in the main text, it is assumed that network losses are negligible and voltage drops and reactive powers are not represented (see, e.g., Bushnell and Stoft 1995; Kirschen and Strbac 2004). Thus, we are allowed to use the linear power flow equations found by dividing the power dispatched from one generation center with respect to the path's total susceptance. Bushnell and Stoft (1995) explained this principle by using admittance. However, admittance and susceptance are equivalent in this context because we are only concerned with the ease of flow on the power lines. Therefore, whether admittance or susceptance is used in this context is not important. This principle is known as superposition principle. In this principle, only one generation center is taken into account at each step and power amounts on the lines are found. At the end, power amounts on the lines are summed algebraically and net power amounts are obtained.

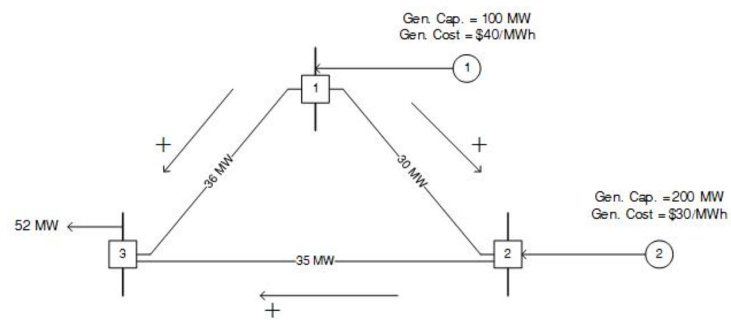

Figure 2.D.1 *Hypothetical directions of power flow*

For our numerical example, we assume that the directions of power flows occur as seen in Figure 2.D.1. Let us first consider generation center 1. If power is dispatched from this center, then it flows in two paths: from center 1 to center 3 and from center 1 through center 2 to center 3. If two power lines are connected serially, then total susceptance is found by (see, e.g., Svoboda and Dorf 2014)

$$\frac{1}{b_{123}} = \frac{1}{b_{12}} + \frac{1}{b_{23}} = \frac{1}{b} + \frac{1}{b} = \frac{2}{b} \Longrightarrow b_{123} = \frac{b}{2} \tag{2D.1}$$

where b_{123} denotes the total susceptance of path from center 1 through center 2 to center 3, b_{12} denotes the susceptance of power line connecting centers 1 and 2, and b_{23} denotes the susceptance of power line connecting centers 2 and 3. Here, we denote one unit of susceptance as b. Let $\mathcal{V}_1$ and $\mathcal{V}_3$ denote the voltage at centers 1 and 3, respectively. By using Ohm's law (see, e.g., Svoboda and Dorf 2014), we can write that

$$\mathcal{V}_1 - \mathcal{V}_3 = \frac{L_{13}}{b_{13}} = \frac{L_{123}}{b_{123}} = \frac{L_{13}}{b} = \frac{L_{123}}{\frac{b}{2}} \tag{2D.2}$$

where L_{13} denotes the amount of power flow from center 1 to center 3 and L_{123} denotes the power flow from center 1 through center 2 to center 3. Therefore, Equation (2D.2), we can say that $2L_{123} = L_{13}$. Hence, the following power flow equations can be written:

$$L_{12} = \frac{1}{3}G_1, \;\; L_{13} = \frac{2}{3}G_1, \;\; L_{23} = \frac{1}{3}G_1 \tag{2D.3}$$

Let us now consider the second generation center. If power is dispatched from this center, then it flows in two paths: from center 2 through center 1 to center 3 and from center 2 to center 3. By using the Ohm's law, we can write that

$$\mathcal{V}_2 - \mathcal{V}_3 = \frac{L_{23}}{b_{23}} = \frac{L_{213}}{b_{213}} = \frac{L_{23}}{b} = \frac{L_{213}}{\frac{b}{2}} \Rightarrow L_{23} = 2L_{213} \tag{2D.4}$$

Hence, power flow equations can be written in the following form:

$$L_{12} = -\frac{1}{3}G_2, \;\; L_{13} = \frac{1}{3}G_2, \;\; L_{23} = \frac{2}{3}G_2 \tag{2D.5}$$

By summing up these power flows, one can reach the net power amounts as follows:

$$L_{12} = \frac{1}{3}G_1 - \frac{1}{3}G_2, L_{13} = \frac{2}{3}G_1 + \frac{1}{3}G_2, L_{23} = \frac{1}{3}G_1 + \frac{2}{3}G_2 \tag{2D.6}$$

We note that net power flows on the line connecting centers 1 and 2 have reverse directions. Thus, the signs of these power flows are reverse. In fact, the power flows on the line connecting centers 1 and 2 do not cancel each other. Rather, the superposition principle is used to find the actual power flow on the line. Thus, we can call the individual power amounts triggered by each generation center fictitious (see, e.g., Kuphaldt 2006).

Thermal limit constraints of the power lines and capacity constraints for the generation centers are added:

$$-30 \leq L_{12} \leq 30 \tag{2D.7}$$

$$-36 \leq L_{13} \leq 36 \tag{2D.8}$$

$$-35 \leq L_{23} \leq 35 \tag{2D.9}$$

$$0 \leq G_1 \leq 100 \tag{2D.10}$$

$$0 \leq G_2 \leq 200 \tag{2D.11}$$

Because we do not know the right directions of power flows on the lines, we add the capacities of the power lines with both negative and positive signs. Finally, the demand amount should be equal to the total amount of power dispatched. Thus, as a final equation, we add the following constraint:

$$G_1 + G_2 = 52 \tag{2D.12}$$

The objective of the OPF problem is to minimize total generation costs. Thus, the objective function is

$$\min_{G_1, G_2} 40G_1 + 30G_2 \tag{2D.13}$$

Appendix 2.E LMP (Without Investment)

We recall that a demand value is denoted by $E_{(t,k)}$ at the state (t,k).

$\boldsymbol{E_{(2,1)} = 59.22\ MW}$

Solution of the OPF problem: We first consider the cheapest generation center (generation center 2). If all 59.22 MW is dispatched from this center, then $L_{23} = 39.48\ MW$, $L_{12} = -19.74\ MW$ and $L_{13} = 19.74\ MW$. However, $L_{23} > \overline{L_{23}}$. Thus, we have to increase the dispatch amount of generation center 1 and simultaneously decrease the dispatch amount of generation center 2. Let ΔG_1 and ΔG_2 be the changes in dispatches of generation centers 1 and 2, respectively. Thus, it should be $\Delta G_1 + \Delta G_2 = 0$ and $\frac{1}{3}\Delta G_1 + \frac{2}{3}(59.22 + \Delta G_2) = 35$. The solution of this set of equations is $\Delta G_1 = 13.44$ and $\Delta G_2 = -13.44$. Thus, $G_1 = 13.44\ MW$ and $G_2 = 45.78\ MW$. Since the power flows on the other lines resulting from this dispatch do not violate the capacity limits, we can say that this is the optimal solution.

The LMP at center 1: In order to calculate the LMP at center 1, we increase the load amount by 1 MW at this center. After that, we first check the cheapest generation center to supply this additional load. If the dispatch amount of this center is increased by 1 MW, then $\frac{2}{3}$ MW power flows from center 2 to center 1. In this case, $L_{23} = 35.33\ MW$ which violates $\overline{L_{23}}$. Thus, we check the second cheapest generation center in order to supply 1 MW additional load. Since remaining capacity of this center is sufficient for supplying, then this center is dispatched. The change in total system cost is \$40/h and the LMP at center 1 is \$40/MWh.

The LMP at center 2: We increase load amount by 1 MW at center 2. The cheapest generation center should be checked first to supply 1 MW load. Since the remaining capacity of this center is sufficient for supplying, then this center is dispatched. The change in total system cost is \$30/h and the LMP at this center is \$30/MWh.

The LMP at center 3: The load amount at this center is increased by 1 MW. We check the cheapest generation center first. It is observed that if 1 MW load is supplied by this center, then $L_{23} = 35.66$. Since $L_{23} > \overline{L_{23}}$, we cannot dispatch generation center 2 on its own. Secondly, we have to check the first generation center to supply 1 MW load. If the dispatch of this center is increased by 1 MW, then $L_{23} = 35.33$, which also violates $\overline{L_{23}}$. Then, it means that we cannot dispatch this generation center by its own. At this point, we find a combinational dispatch of the centers. Let ΔG_1 and ΔG_2 be the changes in dispatches of generation centers 1 and 2, respectively. Then, $\Delta G_1 + \Delta G_2 = 1$ and $\frac{1}{3}\Delta G_1 + \frac{2}{3}\Delta G_2 = 0$ where the first equation represents that change in total dispatch should be equal to 1 MW additional demand and the second equation represents that power flow on the line connecting centers 2 and 3 must stay at 35 MW. If we solve this set of equations, we get $\Delta G_1 = 2$ and $\Delta G_2 = -1$. Thus, the change in total system cost is $2MW \cdot \$40/MWh - 1MW \cdot \$30/MWh = \$50/h$ and the LMP at this center is \$50/MWh.

Network Revenue: In summary, at the end of all these calculations, the following values are obtained regarding the LMPs at each center, demand value and the dispatch amounts of the generation centers: $\pi_1 = 40$, $\pi_2 = 30$, $\pi_3 = 50$, $D_3 = 59.22$, $G_1 = 13.44$ and $G_2 = 45.78$. By using Equation (2.1) and these values, network revenue denoted by $NR_{(2,1)}$ in the main text is calculated as \$1050/h.

$\boldsymbol{E_{(2,2)} = 45.66\ MW}$

Solution of the OPF problem: We first consider the cheapest generation center. If all 45.66 MW is dispatched from this center, then $L_{23} = 30.44\ MW$, $L_{12} = -15.22\ MW$ and $L_{13} = 15.22\ MW$. Since none of these power flows violates the capacity limits of the corresponding power lines, this is accepted as optimal solution. Thus, at optimality, $G_1 = 0\ MW$ and $G_2 = 45.66\ MW$.

The LMP at center 1: We increase the load amount by 1 MW at center 1. After that, we first check the cheapest generation center to supply this additional load. If the dispatch amount of this center is increased by 1 MW, then $\frac{1}{3}$ MW flows from center 2 through center 3 to center 1. Additionally, $\frac{2}{3}$ MW power flows from center 2 to center 1 directly. In this case, $L_{12} = -15.89$, $L_{13} = 14.89$ and $L_{23} = 30.77$. Since none of these values violates the capacity limits of the corresponding power lines, generation center 2 can be dispatched to supply the additional load at center 1. Thus, the change in total system cost is \$30/h and the LMP at center 1 is \$30/MWh.

The LMP at center 2: We increase load amount by 1 MW at center 2. The cheapest generation center should be checked first to supply 1 MW load. Since the remaining capacity of this center is sufficient for supplying, then this center is dispatched. The change in total system cost is \$30/h and the LMP at this center is \$30/MWh.

The LMP at center 3: The load amount at this center is increased by 1 MW. We first check the cheapest generation center. It is observed that if 1 MW load is supplied by this center, then $L_{23} = 31.11\ MW$, $L_{12} = -15.55\ MW$ and $L_{13} = 15.55$. Since none of these violates the capacity limits of the corresponding power lines, generation center 2 can be dispatched to supply the additional load at center 3. Thus, the change in total system cost is \$30/h and the LMP at this center is \$30/MWh.

Network Revenue: In summary, at the end of all these calculations, the following values are obtained regarding the LMPs at each center, demand value and the dispatch amounts of the generation centers: $\pi_1 = 30$, $\pi_2 = 30$, $\pi_3 = 30$, $D_3 = 45.66$, $G_1 = 0$ and $G_2 = 45.66$. By using Equation (2.1) and these values, network revenue denoted by $NR_{(2,2)}$ in the main text is calculated as \$0/h.

$\boldsymbol{E_{(1,1)} = 52\ MW}$

Solution of the OPF problem: We first consider the cheapest generation center. If all 52 MW is dispatched from this center, then $L_{23} = 34.67\ MW$, $L_{12} = -17.33\ MW$ and $L_{13} = 17.33\ MW$. Since none of these power flows violates the capacity limits of the corresponding power lines, this is accepted as optimal solution. Thus, at optimality, $G_1 = 0\ MW$ and $G_2 = 52\ MW$.

The LMP at center 1: We increase the load amount by 1 MW at center 1. After that, we first check the cheapest generation center to supply this additional load. If the dispatch amount of this center is increased by 1 MW, then $\frac{1}{3}$ MW flows from center 2 through center 3 to center 1. Additionally, $\frac{2}{3}$ MW power flows from center 2 to center 1 directly. In this case, $L_{12} = -18\ MW$, $L_{13} = 17\ MW$ and $L_{23} = 35\ MW$. Since none of these values violates the capacity limits of the corresponding power lines, generation center 2 can be dispatched to supply the additional load at center 1. Thus, the change in total system cost is \$30/h and the LMP at center 1 is \$30/MWh.

The LMP at center 2: We increase load amount by 1 MW at center 2. The cheapest generation center should be checked first to supply 1 MW load. Since the remaining capacity of this center is sufficient for supplying, then this center is dispatched. The change in total system cost is \$30/h and the LMP at this center is \$30/MWh.

The LMP at center 3: The load amount at this center is increased by 1 MW. We first check the cheapest generation center. It is observed that if 1 MW load is supplied by this center, then $\frac{2}{3}$ MW additional power flows on the line from center 2 to center 3. Thus, $L_{23} = 35.33\ MW$. Since $L_{23} > \overline{L_{23}}$, we cannot dispatch generation center 2 on its own. Secondly, we have to check the first generation center to supply 1 MW load. If the dispatch of this center is increased by 1 MW, then $L_{23} = 35\ MW \leq \overline{L_{23}}$. Thus, additional load at center 3 can be supplied from generation center 1. The change in total system cost is \$40/h and the LMP at this center is \$40/MWh.

Network Revenue: In summary, at the end of all these calculations, the following values are obtained regarding the LMPs at each center, demand value and the dispatch amount of the generation centers: $\pi_1 = 30$, $\pi_2 = 30$, $\pi_3 = 40$, $D_3 = 52$, $G_1 = 0$ and $G_2 = 52$. By using Equation (2.1) and these values, network revenue denoted by $NR_{(1,1)}$ in the main text is calculated as \$520/h.

Appendix 2.F OPF (Investment Between Centers 1 And 3)

Total susceptance of the power lines connecting centers 1 and 3 is doubled because they are connected parallel. Thus, it becomes $2b$. Let's consider the first generation center. We are able to write the following equations by using Ohm's law.

$$\mathcal{V}_1 - \mathcal{V}_3 = \frac{L_{13}}{b_{13}} = \frac{L_{123}}{b_{123}} = \frac{L_{13}}{2b} = \frac{L_{123}}{\frac{b}{2}} \Longrightarrow L_{13} = 4L_{123} \tag{2F.1}$$

$$L_{12} = \frac{1}{5}G_1, \;\; L_{13} = \frac{4}{5}G_1, \;\; L_{23} = \frac{1}{5}G_1 \tag{2F.2}$$

Let's now consider the second generation center. It is critical to find the total susceptance on the path from centers 2 to 1 to 3. We know that

$$\frac{1}{b_{213}} = \frac{1}{b_{21}} + \frac{1}{b_{13}} = \frac{1}{b} + \frac{1}{2b} = \frac{3}{2b} \Longrightarrow b_{213} = \frac{2b}{3} \tag{2F.3}$$

Thus,

$$V_2 - V_3 = \frac{L_{23}}{b_{23}} = \frac{L_{213}}{b_{213}} = \frac{L_{23}}{b} = \frac{L_{213}}{\frac{2b}{3}} \Rightarrow 2L_{23} = 3L_{213} \tag{2F.4}$$

$$L_{12} = -\frac{2}{5}G_2, \;\; L_{13} = \frac{2}{5}G_2, \;\; L_{23} = \frac{3}{5}G_2 \tag{2F.5}$$

Therefore, net power flow equations can be written as follows:

$$L_{12} = \frac{1}{5}G_1 - \frac{2}{5}G_2, L_{13} = \frac{4}{5}G_1 + \frac{2}{5}G_2\,, L_{23} = \frac{1}{5}G_1 + \frac{3}{5}G_2 \tag{2F.6}$$

The rest of constraints are capacity limits of generators and thermal limits of power lines, given in Equations (2D.7) – (2D.11). We note that $-40 \le L_{13} \le 40$ because a 4 MW power line is added. Furthermore, an equation representing the equality between demand amount and total dispatch should be added.

Appendix 2.G LMP (Investment Between Centers 1 And 3)

$\boldsymbol{E_{(21)} = 59.22\ MW}$

Solution of the OPF problem: We consider at first the cheapest generation center. If all 59.22 MW is dispatched from this center, then $L_{23} = 35.53\ MW$, $L_{12} = -23.69\ MW$ and $L_{13} = 23.69\ MW$. However, $L_{23} > \overline{L_{23}}$. Thus, we have to increase the dispatch amount of generation center 1 and simultaneously decrease the dispatch amount of generation center 2. Let ΔG_1 and ΔG_2 be the change amount in dispatch of centers 1 and 2, respectively. Thus, it should be $\Delta G_1 + \Delta G_2 = 0$ and $\frac{1}{5}\Delta G_1 + \frac{3}{5}(59.22 + \Delta G_2) = 35$. The solution of this set of equations are $\Delta G_1 = 1.33$ and $\Delta G_2 = -1.33$. Thus, $G_1 = 1.33\ MW$ and $G_2 = 57.89\ MW$. Since the power flows on the other lines resulting from the dispatch do not violate the capacity limits, we can say that this is the optimal solution.

The LMP at center 1: We increase the load amount at center 1. After that, we check the cheapest generation center at first to supply this additional load. If the dispatch amount of this center is increased by 1 MW, then $\frac{2}{5}$ MW additional power flows from centers 2 to 3, which means $L_{23} = 35.93 > \overline{L_{23}}$. Thus, we check the second cheapest generation center in order to supply 1 MW additional load. Since remaining capacity of this generation center is sufficient for supplying, it is dispatched. The change in total system cost is \$ 40/h, and thus, the LMP at center 1 is \$40/MWh.

The LMP at center 2: We increase load amount at center 2. The cheapest generation center should be checked at first to supply 1 MW load. Since the remaining capacity of this center is sufficient for supplying, it is dispatched. The change in total system cost is \$ 30/h and the LMP at this center is \$30/MWh.

The LMP at center 3: The load amount at this center is increased by 1 MW. We check the cheapest generation center at first. It is observed that if 1 MW load is supplied by this center, then $\frac{3}{5}$ MW additional power flows from centers 2 to 3, which means $L_{23} = 36.13 > \overline{L_{23}}$. It means that we cannot dispatch the generation center 2 on its own. Secondly, we have to check the first generation center to supply 1 MW load. If the dispatch of this center is increased by 1 MW, then $\frac{1}{5}$ MW additional power flows from centers 2 to 3, which means $L_{23} = 35.73 > \overline{L_{23}}$. Hence, we cannot dispatch this center by its own.

At this point, we find a combinational dispatch of the generation centers. Let ΔG_1 and ΔG_2 be the changes in dispatch of the centers 1 and 2, respectively. Then, $\Delta G_1 + \Delta G_2 = 1$ and $\frac{1}{5}\Delta G_1 + \frac{3}{5}\Delta G_2 = 0$ should be satisfied. If we solve this set of equations, we get $\Delta G_1 = 1.5$ and $\Delta G_2 = -0.5$. Thus, the change in total system cost is $1.5MW \cdot \$40/MWh - 0.5MW \cdot \$30/MWh = \$\,45/h$ and the LMP at this center is \$45/MWh.

Network Revenue: In summary, at the end of all these calculations, the following values are obtained regarding the LMPs at each center, demand value and the dispatch amount of the generation centers: $\pi_1 = 40$, $\pi_2 = 30$, $\pi_3 = 45$, $D_3 = 59.22$, $G_1 = 1.33$ and $G_2 = 57.89$. By

using Equation (2.1) and these values, network revenue denoted by $NR_{(2,1)}$ in the main text are calculated as \$875/h.

$\boldsymbol{E_{(2,2)} = 45.66\ MW}$

Solution of the OPF problem: We consider the cheapest generation center at first. If all 45.66 MW is dispatched from this center, then $L_{23} = 27.40\ MW$, $L_{12} = -18.26\ MW$ and $L_{13} = 18.26\ MW$. Since none of the power flows violates the capacity limits of the corresponding power lines, this is accepted as optimal solution. Thus, at optimality, $G_1 = 0\ MW$ and $G_2 = 45.66\ MW$.

The LMP at center 1: We increase the load amount at center 1. After that, we check the cheapest generation center at first to supply this additional load. If the dispatch amount of this center is increased by 1 MW, it can be observed that $L_{23} = 27.80\ MW$, $L_{12} = -18.86\ MW$ and $L_{13} = 17.86\ MW$. Since none of these violates the corresponding capacities, generation center 2 can be dispatched to supply the additional load at center 1. The change in total system cost is \$30/h, and thus, the LMP at center 1 is \$30/MWh.

The LMP at center 2: We increase load amount at center 2. The cheapest generation center should be checked at first to supply 1 MW load. Since the remaining capacity of this center is sufficient for supplying, it is dispatched. The change in total system cost is \$30/h and the LMP at this center is \$30/MWh.

The LMP at center 3: The load amount at this center is increased by 1 MW. We check the cheapest generation center at first. It is observed that if 1 MW load is supplied by this center, then $L_{23} = 28\ MW$, $L_{12} = -18.66\ MW$ and $L_{13} = 18.66\ MW$. Since none of these violates the corresponding capacities, generation center 2 can be dispatched to supply the additional load at center 1. The change in total system cost is \$30/h, and thus, the LMP at center 1 is \$30/MWh.

Network Revenue: In summary, at the end of all these calculations, the following values are obtained regarding the LMPs at each center, demand value and the dispatch amount of the generators: $\pi_1 = 30, \pi_2 = 30, \pi_3 = 30, D_3 = 45.66, G_1 = 0$ and $G_2 = 45.66$. By using Equation (2.1) and these values, network revenue denoted by $NR_{(2,2)}$ in the main text are calculated as \$0/h.

$\boldsymbol{E_{(1,1)} = 52\ MW}$

Solution of the OPF problem: We consider the cheapest generation center at first. If all 52 MW is dispatched from this center, then $L_{23} = 31.20\ MW$, $L_{12} = -20.8\ MW$ and $L_{13} = 20.8\ MW$. Since none of the power flows violates the capacity limits of the corresponding power lines, this is accepted as optimal solution. Thus, at optimality, $G_1 = 0\ MW$ and $G_2 = 52\ MW$.

The LMP at center 1: We increase the load amount at center 1. After that, we check the cheapest generation center at first to supply this additional load. If the dispatch amount of this center is increased by 1 MW, it can be observed that $L_{23} = 31.60\ MW$, $L_{12} = -21.4\ MW$ and $L_{13} = 20.4\ MW$. Since none of these violates the capacity limits, generation center 2 can be dispatched to supply the additional load at center 1. The change in total system cost is \$30/h, and thus, the LMP at center 1 is \$30/MWh.

The LMP at center 2: We increase load amount at center 2. The cheapest generation center should be checked at first to supply 1 MW load. Since the remaining capacity of this center is sufficient for supplying, it is dispatched. The change in total system cost is \$30/h and the LMP at this center is \$30/MWh.

The LMP at center 3: The load amount at this center is increased by 1 MW. We check the cheapest generation center at first. It is observed that if 1 MW load is supplied by this center, $L_{23} = 31.80\ MW$, $L_{12} = -21.20\ MW$ and $L_{13} = 21.20\ MW$. None of these violates the capacity limits; thus, generation center 2 can be dispatched to supply the additional load at center 3. The change in total system cost is \$30/h and the LMP at this center is \$30/MWh.

Network Revenue: In summary, at the end of all these calculations, the following values are obtained regarding the LMPs at each center, demand value and the dispatch amount of the generators: $\pi_1 = 30$, $\pi_2 = 30$, $\pi_3 = 30$, $D_3 = 52$, $G_1 = 0$ and $G_2 = 52$. By using Equation (2.1) and these values, network revenue denoted by $NR_{(1,1)}$ in the main text are calculated as \$0/h.

Appendix 2.H OPF (Investment Between Centers 1 And 2)

Total susceptance of the power lines connecting centers 1 and 2 is doubled because they are connected parallel. Thus, $b_{12} = 2b$. Let's consider the first generation center. It is critical to find the total susceptance on the path from centers 1 to 2 to 3. We derive that

$$\frac{1}{b_{123}} = \frac{1}{b_{12}} + \frac{1}{b_{23}} = \frac{1}{2b} + \frac{1}{b} = \frac{3}{2b} \Longrightarrow b_{123} = \frac{2b}{3} \tag{2H.1}$$

Thus,

$$\mathcal{V}_1 - \mathcal{V}_3 = \frac{L_{13}}{b_{13}} = \frac{L_{123}}{b_{123}} = \frac{L_{13}}{b} = \frac{L_{123}}{\frac{2b}{3}} \Rightarrow 2L_{13} = 3L_{123} \tag{2H.2}$$

$$L_{12} = \frac{2}{5}G_1, \;\; L_{13} = \frac{3}{5}G_1, \;\; L_{23} = \frac{2}{5}G_1 \tag{2H.3}$$

We can write a similar set of equations for generation center 2. That is,

$$\frac{1}{b_{213}} = \frac{1}{b_{21}} + \frac{1}{b_{13}} = \frac{1}{2b} + \frac{1}{b} = \frac{3}{2b} \Longrightarrow b_{213} = \frac{2b}{3} \tag{2H.4}$$

$$V_2 - V_3 = \frac{L_{23}}{b_{23}} = \frac{L_{213}}{b_{213}} = \frac{L_{23}}{b} = \frac{L_{213}}{\frac{2b}{3}} \Rightarrow 2L_{23} = 3L_{213} \tag{2H.5}$$

$$L_{12} = -\frac{2}{5}G_2, \;\; L_{13} = \frac{2}{5}G_2, \;\; L_{23} = \frac{3}{5}G_2 \tag{2H.6}$$

Therefore, net power flow equations can be written as follows:

$$L_{12} = \frac{2}{5}G_1 - \frac{2}{5}G_2, L_{13} = \frac{3}{5}G_1 + \frac{2}{5}G_2, L_{23} = \frac{2}{5}G_1 + \frac{3}{5}G_2 \tag{2H.7}$$

The rest of constraints are capacity limits of generation centers and thermal limits of power lines. We note that $-34 \leq L_{12} \leq 34$ because a 4 MW power line is added. Furthermore, an equation expressing the equality between demand amount and total dispatch should be added.

Appendix 2.I LMP (Investment Between Centers 1 And 2)

$\boldsymbol{E_{(2,1)} = 59.22\ MW}$

Solution of the OPF problem: We consider the cheapest generation center at first. If all 59.22 MW is dispatched from this center, then $L_{23} = 35.53$ MW, $L_{12} = -23.69$ MW and $L_{13} = 23.69$ MW. However, $L_{23} > \overline{L_{23}}$. Thus, we have to increase the dispatch amount of generation center 1 and simultaneously decrease the dispatch amount of generation center 2. Let ΔG_1 and ΔG_2 be the change amount in dispatch of centers 1 and 2, respectively. Thus, it should be $\Delta G_1 + \Delta G_2 = 0$ and $\frac{2}{5}\Delta G_1 + \frac{3}{5}(59.22 + \Delta G_2) = 35$. The solution of this set of equations are $\Delta G_1 = 2.66$ and $\Delta G_2 = -2.66$. Thus, $G_1 = 2.66\ MW$ and $G_2 = 56.56\ MW$. Since the power flows on the other lines resulting from the dispatch do not violate the capacity limits, we can say that this is the optimal solution.

The LMP at center 1: We increase the load amount at center 1. After that, we check the cheapest generation center at first to supply this additional load. If the dispatch amount of this center is increased by 1 MW, then $\frac{1}{5}$ MW additional power flows from centers 2 to 3, which means $L_{23} = 35.2 > \overline{L_{23}}$. Thus, we check the second cheapest generation center in order to supply 1 MW additional load. Since remaining capacity of this center is sufficient for supplying, then this generator is dispatched. The change in total system cost is \$40/h, and thus, the LMP at center 1 is \$40/MWh.

The LMP at center 2: We increase load amount at center 2. The cheapest generation center should be checked to supply 1 MW load. Since the remaining capacity of this center is sufficient for supplying, it is dispatched. The change in total system cost is \$30/h and the LMP at this center is \$30/MWh.

The LMP at center 3: The load amount at this center is increased by 1 MW. We check the cheapest generation center at first. It is observed that if 1 MW load is supplied by this center, then $\frac{3}{5}$ MW additional power flows from centers 2 to 3, which means $L_{23} = 35.60 > \overline{L_{23}}$. It means that we cannot dispatch the generation center 2 on its own. Secondly, we have to check the first generation center to supply 1 MW load. If the dispatch of this center is increased by 1 MW, then $\frac{2}{5}$ MW additional power flows from centers 2 to 3, which means $L_{23} = 35.40 > \overline{L_{23}}$. Hence, we cannot dispatch this center by its own.

At this point, we find a combinational dispatch of the generation centers. Let ΔG_1 and ΔG_2 be the changes in dispatch of the generation centers 1 and 2, respectively. Then, $\Delta G_1 + \Delta G_2 = 1$ and $\frac{2}{5}\Delta G_1 + \frac{3}{5}\Delta G_2 = 0$ should be satisfied. If we solve this set of equations, we get $\Delta G_1 = 3$ and $\Delta G_2 = -2$. Thus, the change in total system cost is $3\ \text{MW} \cdot \$40/\text{MWh} - 2\ \text{MW} \cdot \$30/\text{MWh} = \$60/\text{h}$ and the LMP at this center is \$60/MWh.

Network Revenue: In summary, at the end of all these calculations, the following values are obtained regarding the LMPs at each center, demand value and the dispatch amount of the generation centers: $\pi_1 = 40$, $\pi_2 = 30$, $\pi_3 = 60$, $D_3 = 59.22$, $G_1 = 2.66$ and $G_2 = 56.56$. By

using Equation (2.1) and these values, network revenue denoted by $NR_{(2,1)}$ in the main text are calculated as \$1750/h.

$\boldsymbol{E_{(2,2)} = 45.66\ MW}$

Solution of the OPF problem: We consider the cheapest generation center at first. If all 45.66 MW is dispatched from this center, then $L_{23} = 27.40\ MW$, $L_{12} = -18.26\ MW$ and $L_{13} = 18.26\ MW$. Since none of the power flows violates the capacity limits of the corresponding power lines, this is accepted as optimal solution. Thus, at optimality, $G_1 = 0\ MW$ and $G_2 = 45.66\ MW$.

The LMP at center 1: We increase the load amount at center 1. After that, we check the cheapest generation center at first to supply this additional load. If the dispatch amount of this center is increased by 1 MW, it can be observed that $L_{23} = 27.60\ MW$, $L_{12} = -19.06\ MW$ and $L_{13} = 18.06\ MW$. Since none of these violates the corresponding capacities, generation center 2 can be dispatched to supply the additional load at center 1. The change in total system cost is \$30/h, and thus, the LMP at center 1 is \$30/MWh.

The LMP at center 2: We increase load amount at center 2. At first, the cheapest generation center should be checked to supply 1 MW load. Since the remaining capacity of this center is sufficient for supplying, it is dispatched. The change in total system cost is \$30/h and the LMP at this center is \$30/MWh.

The LMP at center 3: The load amount at this center is increased by 1 MW. We check the cheapest generation center at first. It is observed that if 1 MW load is supplied by this center, then $L_{23} = 28\ MW$, $L_{12} = -18.66\ MW$ and $L_{13} = 18.66\ MW$. Since none of these violates the corresponding capacities, generation center 2 can be dispatched to supply the additional load at center 1. The change in total system cost is \$30/h, and thus, the LMP at center 3 is \$30/MWh.

Network Revenue: In summary, at the end of all these calculations, the following values are obtained regarding the LMPs at each center, demand value and the dispatch amount of the generators: $\pi_1 = 30, \pi_2 = 30, \pi_3 = 30, D_3 = 45.66, G_1 = 0$ and $G_2 = 45.66$. By using Equation (2.1) and these values, network revenue denoted by $\mathrm{NR}_{(2,2)}$ in the main text are calculated as \$0/h.

$\boldsymbol{E_{(1,1)} = 52\ MW}$

Solution of the OPF problem: We consider the cheapest generation center at first. If all 52 MW is dispatched from this center, then $L_{23} = 31.20\ MW$, $L_{12} = -20.80\ MW$ and $L_{13} = 20.80\ MW$. Since none of the power flows violates the capacity limits of the corresponding power lines, this is accepted as optimal solution. Thus, at optimality, $G_1 = 0\ MW$ and $G_2 = 52\ MW$.

The LMP at center 1: We increase the load amount at center 1. After that, we check the cheapest generation center to supply this additional load. If the dispatch amount of this center is increased by 1 MW, it can be observed that $L_{23} = 31.40\ MW$, $L_{12} = -21.60\ MW$ and $L_{13} = 20.60\ MW$. Since none of these violates the capacity limits, generation center 2 can be dispatched to supply the additional load at center 1. The change in total system cost is \$30/h, and thus, the LMP at center 1 is \$30/MWh.

The LMP at center 2: We increase load amount at center 2. At first, the cheapest generation center should be checked to supply 1 MW load. Since the remaining capacity of this center is sufficient for supplying, it is dispatched. The change in total system cost is \$30/h and the LMP at this center is \$30/MWh.

The LMP at center 3: The load amount at this center is increased by 1 MW. We check the cheapest generation center at first. It is observed that if 1 MW load is supplied by this center, $L_{23} = 31.80\ MW$, $L_{12} = -21.20\ MW$ and $L_{13} = 21.20\ MW$. None of these violates the capacity limits; thus, generation center 2 can be dispatched to supply the additional load at center 3. The change in total system cost is \$30/h and the LMP at this center is \$30/MWh.

Network Revenue: In summary, at the end of all these calculations, the following values are obtained regarding the LMPs at each center, demand value and the dispatch amount of the generation centers: $\pi_1 = 30$, $\pi_2 = 30$, $\pi_3 = 30$, $D_3 = 52$, $G_1 = 0$ and $G_2 = 52$. By using Equation (2.1) and these values, network revenue denoted by $NR_{(1,1)}$ in the main text are calculated as \$0/h.

Appendix 2.J OPF (Investment Between Centers 2 And 3)

Total susceptance of the power lines connecting centers 2 and 3 is doubled because they are connected parallel. Thus, $b_{23} = 2b$. Let's consider the first generation center. It is critical to find the total susceptance on the path from centers 1 to 2 to 3. We know that

$$\frac{1}{b_{123}} = \frac{1}{b_{12}} + \frac{1}{b_{23}} = \frac{1}{b} + \frac{1}{2b} = \frac{3}{2b} \Longrightarrow b_{123} = \frac{2b}{3} \tag{2J.1}$$

Thus,

$$\mathcal{V}_1 - \mathcal{V}_3 = \frac{L_{13}}{b_{13}} = \frac{L_{123}}{b_{123}} = \frac{L_{13}}{b} = \frac{L_{123}}{\frac{2b}{3}} \Rightarrow 2L_{13} = 3L_{123} \tag{2J.2}$$

$$L_{12} = \frac{2}{5}G_1, \;\; L_{13} = \frac{3}{5}G_1, \;\; L_{23} = \frac{2}{5}G_1 \tag{2J.3}$$

We can write a similar set of equations for generation center 2. That is,

$$\mathcal{V}_2 - \mathcal{V}_3 = \frac{L_{23}}{b_{23}} = \frac{L_{213}}{b_{213}} = \frac{L_{23}}{2b} = \frac{L_{213}}{\frac{b}{2}} \Rightarrow L_{23} = 4L_{213} \tag{2J.4}$$

$$L_{12} = -\frac{1}{5}G_2, \;\; L_{13} = \frac{1}{5}G_2, \;\; L_{23} = \frac{4}{5}G_2 \tag{2J.5}$$

Therefore, net power flow equations can be written as follows:

$$L_{12} = \frac{2}{5}G_1 - \frac{1}{5}G_2, L_{13} = \frac{3}{5}G_1 + \frac{1}{5}G_2, L_{23} = \frac{2}{5}G_1 + \frac{4}{5}G_2 \tag{2J.6}$$

The rest of constraints are capacity limits of generation centers and thermal limits of power lines. We note that $-39 \leq L_{23} \leq 39$ because a 4 MW power line is added. Furthermore, an equation expressing the equality between demand amount and total dispatch should be added.

Appendix 2.K LMP (Investment Between Centers 2 And 3)

$\boldsymbol{E_{(2,1)} = 59.22\ MW}$

Solution of the OPF problem: We consider the cheapest generation center. If all 59.22 MW is dispatched from this center, then $L_{23} = 47.38\ MW$, $L_{12} = -11.84\ MW$ and $L_{13} = 11.84\ MW$. However, $L_{23} > \overline{L_{23}}$. Thus, we have to increase the dispatch amount of generation center 1 and simultaneously decrease the dispatch amount of generation center 2. Let ΔG_1 and ΔG_2 be the change amount in dispatch of generation centers 1 and 2, respectively. Thus, it should be $\Delta G_1 + \Delta G_2 = 0$ and $\frac{2}{5}\Delta G_1 + \frac{4}{5}(59.22 + \Delta G_2) = 35$. The solution of this set of equations are $\Delta G_1 = 30.94$ and $\Delta G_2 = -30.94$. Thus, $G_1 = 30.94\ MW$ and $G_2 = 28.28\ MW$. Since the power flows on the other lines resulting from the dispatch do not violate the capacity limits, we can say that this is the optimal solution.

The LMP at center 1: We increase the load amount at center 1. After that, we check the cheapest generation center at first to supply this additional load. If the dispatch amount of this center is increased by 1 MW, then $\frac{2}{5}$ MW additional power flows from centers 2 to 3, which means $L_{23} = 35.4 > \overline{L_{23}}$. Thus, we check the second cheapest generation center in order to supply 1 MW additional load. Since remaining capacity of this center is sufficient for supplying, it is dispatched. The change in total system cost is \$40/h, and thus, the LMP at center 1 is \$40/MWh.

The LMP at center 2: We increase load amount at center 2. The cheapest generation center should be checked to supply 1 MW load. Since the remaining capacity of this center is sufficient for supplying, it is dispatched. The change in total system cost is \$30/h and the LMP at this center is \$30/MWh.

The LMP at center 3: The load amount at this center is increased by 1 MW. We check the cheapest generation center at first. It is observed that if 1 MW load is supplied by this center, then $\frac{4}{5}$ MW additional power flows from centers 2 to 3, which means $L_{23} = 35.80 > \overline{L_{23}}$. It means that we cannot dispatch the generation center 2 on its own. Secondly, we have to check the first generation center to supply 1 MW load. If the dispatch of this center is increased by 1 MW, then $\frac{2}{5}$ MW additional power flows from centers 2 to 3, which means $L_{23} = 35.40 > \overline{L_{23}}$. Hence, we cannot dispatch this center by its own.

At this point, we find a combinational dispatch of the generation centers. Let ΔG_1 and ΔG_2 be the changes in dispatch of the generation centers 1 and 2, respectively. Then, $\Delta G_1 + \Delta G_2 = 1$ and $\frac{2}{5}\Delta G_1 + \frac{4}{5}\Delta G_2 = 0$ should be satisfied. If we solve this set of equations, we get $\Delta G_1 = 2$ and $\Delta G_2 = -1$. Thus, the change in total system cost is $2\ MW \cdot \$40/MWh - 1\ MW \cdot \$30/MWh = \$50/h$ and the LMP at this center is \$50/MWh.

Network Revenue: In summary, at the end of all these calculations, the following values are obtained regarding the LMPs at each center, demand value and the dispatch amount of the generation centers: $\pi_1 = 40$, $\pi_2 = 30$, $\pi_3 = 50$, $D_3 = 59.22$, $G_1 = 20.94$ and $G_2 = 38.28$. By

using Equation (2.1) and these values, network revenue denoted by $NR_{(2,1)}$ in the main text are calculated as \$975/h.

$\boldsymbol{E_{(2,2)} = 45.66\ MW}$

Solution of the OPF problem: We consider the cheapest generation center. If all 45.66 MW is dispatched from this center, then $L_{23} = 36.53\ MW$, $L_{12} = -9.13\ MW$ and $L_{13} = 9.13\ MW$. Since none of the power flows violates the capacity limits of the corresponding power lines, this is accepted as optimal solution. Thus, at optimality, $G_1 = 0\ MW$ and $G_2 = 45.66\ MW$.

The LMP at center 1: We increase the load amount at center 1. After that, we check the cheapest generation center at first to supply this additional load. If the dispatch amount of this center is increased by 1 MW, it can be observed that $L_{23} = 36.93\ MW$, $L_{12} = -9.73\ MW$ and $L_{13} = 8.73\ MW$. Since none of these violates the corresponding capacities, generation center 2 can be dispatched to supply the additional load at center 1. The change in total system cost is \$30/h, and thus, the LMP at center 1 is \$ 30/MWh.

The LMP at center 2: We increase load amount at center 2. The cheapest generation center should be checked at first to supply 1 MW load. Since the remaining capacity of this center is sufficient for supplying, it is dispatched. The change in total system cost is \$30/h and the LMP at this center is \$30/MWh.

The LMP at center 3: The load amount at this center is increased by 1 MW. We check the cheapest generation center at first. It is observed that if 1 MW load is supplied by this center, then $L_{23} = 37.33\ MW$, $L_{12} = -9.33\ MW$ and $L_{13} = 9.33\ MW$. Since none of these violates the corresponding capacities, generation center 2 can be dispatched to supply the additional load at center 1. The change in total system cost is \$30/h, and thus, the LMP at center 1 is \$30/MWh.

Network Revenue: In summary, at the end of all these calculations, the following values are obtained regarding the LMPs at each center, demand value and the dispatch amount of the generation centers: $\pi_1 = 30$, $\pi_2 = 30$, $\pi_3 = 30$, $D_3 = 45.66$, $G_1 = 0$ and $G_2 = 45.66$. By using Equation (2.1) and these values, network revenue denoted by $NR_{(2,2)}$ in the main text are calculated as \$0/h.

$\boldsymbol{E_{(1,1)} = 52\ MW}$

Solution of the OPF problem: We consider the cheapest generation center at first. If all 52 MW is dispatched from this center, then $L_{23} = 41.60\ MW$, $L_{12} = -10.40\ MW$ and $L_{13} = 10.40\ MW$. However, $L_{23} > \overline{L_{23}}$. Thus, we have to increase the dispatch amount of generation center 1 and simultaneously decrease the dispatch amount of generation center 2. Let ΔG_1 and ΔG_2 be the change amount in dispatch of generation centers 1 and 2, respectively. Thus, it should be $\Delta G_1 + \Delta G_2 = 0$ and $\frac{2}{5}\Delta G_1 + \frac{4}{5}(52 + \Delta G_2) = 39$. The solution of this set of equations are $\Delta G_1 = 6.5$ and $\Delta G_2 = -6.5$. Thus, $G_1 = 6.5\ MW$ and $G_2 = 45.5\ MW$. Since the power flows on the other lines resulting from the dispatch do not violate the capacity limits, we can say that this is the optimal solution.

The LMP at center 1: We increase the load amount at center 1. After that, we check the cheapest generation center at first to supply this additional load. If the dispatch amount of this center is increased by 1 MW, then $\frac{2}{5}$ MW additional power flows from centers 2 to 3, which means $L_{23} = 39.4 > \overline{L_{23}}$. Thus, we check the second cheapest generation center in order to supply 1 MW additional load. Since remaining capacity of this center is sufficient for supplying, it is dispatched. The change in total system cost is \$40/h, and thus, the LMP at center 1 is \$40/MWh.

The LMP at center 2: We increase load amount at center 2. The cheapest generation center should be checked at first to supply 1 MW load. Since the remaining capacity of this center is sufficient for supplying, it is dispatched. The change in total system cost is \$30/h and the LMP at this center is \$30/MWh.

The LMP at center 3: The load amount at this center is increased by 1 MW. We check the cheapest generation center at first. It is observed that if 1 MW load is supplied by this center, then $\frac{4}{5}$ MW additional power flows from centers 2 to 3, which means $L_{23} = 39.80 > \overline{L_{23}}$. It means that we cannot dispatch the generation center 2 on its own. Secondly, we have to check the first generation center to supply 1 MW load. If the dispatch of this center is increased by 1 MW, then $\frac{2}{5}$ MW additional power flows from centers 2 to 3, which means $L_{23} = 39.40 > \overline{L_{23}}$. Hence, we cannot dispatch this center by its own.

At this point, we find a combinational dispatch of the generation centers. Let ΔG_1 and ΔG_2 be the changes in dispatch of the generation centers 1 and 2, respectively. Then, $\Delta G_1 + \Delta G_2 = 1$ and $\frac{2}{5}\Delta G_1 + \frac{4}{5}\Delta G_2 = 0$ should be satisfied. If we solve this set of equations, we get $\Delta G_1 = 2$ and $\Delta G_2 = -1$. Thus, the change in total system cost is $2\ MW \cdot \$40/MWh - 1\ MW \cdot \$30/MWh = \$50/h$ and the LMP at this center is \$50/MWh.

Network Revenue: In summary, at the end of all these calculations, the following values are obtained regarding the LMPs at each center, demand value and the dispatch amount of the generation centers: $\pi_1 = 40$, $\pi_2 = 30$, $\pi_3 = 50$, $D_3 = 52$, $G_1 = 6.5$ and $G_2 = 45.5$. By using Equation (2.1) and these values, network revenue denoted by $NR_{(2,1)}$ in the main text are calculated as \$975/h.

CHAPTER 3. VALUATION OF JUMBOIZATION FOR MILITARY TRANSPORTATION SHIPS: A REAL OPTIONS APPROACH

Introduction

In recent years, a new trend has emerged in industrial practice of engineering design as well as in academic researches. Real options 'in' design have been called by some research practitioners (see, e.g., Wang 2005) to point out that initial design of a product can be accomplished in such a way that the user can modify the product design later with relatively less cost. In other words, while incurring an upfront cost in initial design, the user purchases an option to change the design in future with a relatively lower cost. There exist several real-life examples for this notion such as flexible building for parking (De Neufville et al. 2006) and communications satellite (De Weck et al. 2004).

Ship design is one of the practical areas in which real options 'in' design can be addressed. Jumboization can be listed as one kind of modularity in ship design (see, e.g., Doerry 2014). Jumboization is defined as increasing the capacity of an existing ship by extending its length at a future date. When the decision maker (throughout this chapter, we talk about a single unit as the decision maker although ship design decisions are carried out by several people in reality) decides to execute it, ship's hull is cut into two components, newly built mid-section is inserted and whole process ends with welding of separated hull sections. Jumboization fits to the definition of real options in engineering design because the decision maker needs to pay an upfront cost during initial design to have stronger hull structure by more advanced scantlings than initially required (Buxton and Stephenson 2001). Moreover, the decision maker has the right, but not obligation, to insert the mid-body to the ship. Therefore, upfront cost can be regarded as option premium, which is initially paid to have the option, and jumboization cost can be viewed as strike price in the language of financial options.

In this study, we attempt to evaluate jumboization operations in U.S. Navy ships with real options approach to determine the expected time of jumboization and its value as well as to provide a managerial guideline regarding the choice between fixed (the ship is not designed initially envisioning future jumboization investment) and flexible design (the ship is designed initially envisioning future jumboization investment). In the case of flexible design, jumboization can be conducted more easily and thus less costly.

A careful investigation of jumboization practices in the U.S. Navy reveals that generally replenishment oilers have been jumboized, whose primary purpose is to transport fuel to U.S. Navy ships at sea. Therefore, we build our mathematical model upon replenishment oilers to evaluate jumboization operations. To the best of our knowledge, the U.S. Navy have jumboized 13 ships so far and these ships were not specifically designed initially for jumboization. Hence, Doerry (2014) arises research questions as to what would happen and would there be any cost saving if they were initially designed for jumboization. Moreover, he states that there is a need for analytically rigorous methods to evaluate the flexibilities in design of U.S. Navy ships, which motivates us to conduct this study.

The rest of the chapter is organized as follows: The following section shows the relevant literature, which exemplifies jumboization of ships in public sector ships and types of modularity for U.S. Navy ships. After that, we present the mathematical model consisting of both analytical framework and reconciling discrete counterpart. It is followed by sensitivity analysis uncovering several managerial insights. We propose a managerial guideline in succeeding section concerning the choice between flexible versus fixed design. Then, in order to exhibit the key components of our framework, we solve a numerical example based on a real replenishment oiler. We discuss possible generalizations of some assumptions that we make in the following section of numerical example. At the end, we conclude the chapter by summarizing the key results.

Literature Review

This study contributes to various streams of literature, some of which are reviewed below: Evaluation of jumboization has been conducted for public sector ships in recent years and there is still a growing number of research in this area. For instance, Bačkalov et al. (2014) study the economic feasibility of lengthening of inland vessels in Europe by focusing on two particular reference ships. It is proven that lengthening of larger ships is more attractive than smaller vessels because payback periods are shown to be relatively shorter for larger ships. Ericson and Lake (2014) determine a payback period by considering investment cost and additional income resulting from increased cargo capacity of an example ship. They reveal that lengthening brings about a reduction in required propelling power per cargo ton at a constant speed. Buxton and Stephenson (2001) conduct simulation analysis to evaluate different design strategies for a container ship. Flexible design is proven the most preferable in terms of net present values of the design strategies. Another simulation study is conducted by Knight and Singer (2012) to determine the value of jumboization in a container ship by modeling the freight rate as the underlying stochastic parameter.

On the other hand, to the best of our knowledge, we have not seen any study evaluating the jumboization operations for U.S. Navy ships. Yet there are some researches highlighting real option applications to evaluate modularity concept for U.S. Navy ships. Gregor (2003) assesses flexibilities in naval ship design and procurement. The way of utilizing real options approach is demonstrated in a case study, which emphasizes other characteristics of modular design for the ships rather than jumboization. Page (2012) presents a case study based on a destroyer type ship and discusses the results regarding the financial benefits of modularity. Knight (2014) develops a novel approach comprising of real options approach, utility theory and game theory in order to evaluate the design flexibilities in naval ship design. Case studies focusing on other aspects of modularity rather than jumboization of ships are solved to demonstrate how the proposed approach is conducted.

In the next section, we present our modeling assumptions and analytical framework to evaluate the jumboization option on replenishment oilers.

Mathematical Model

The U.S. Navy possesses several replenishment oilers, which serve in different regions of seas and oceans. We therefore make simplifying assumptions to build the most fundamental model and facilitate the derivation of managerial insights. Our model is based on the following scenario and assumptions: Suppose the decision maker wants to purchase a new replenishment oiler. He/she is requested to choose between two design alternatives; fixed design or flexible design.

Assumption 1: Demand for fuel (tons at a time, e.g. half a month as a unit time interval) by the ships in need of fuel replenishment (in literature, these ships are generally called the receiving ships or the customer ships. We will henceforth use the term 'the receiving ships' to point those ships) follows GBM process, which is mathematically stated as:

$$dD_t = \alpha D_t dt + \sigma D_t dz \tag{3.1}$$

where dz is a Brownian increment; i.e., $dz = \epsilon\sqrt{dt}$, $\epsilon \sim N(0,1)$. In this case, $E[dz] = 0$ and $Var[dz] = dt$.

α (%/unit time) and σ (%/unit time) are defined as growth and volatility parameters of demand evolution. Note that the receiving ships call for fuel replenishment in each unit time, which can be set as a couple of days or a couple of years.

Demand, D_t, is monitored by the decision maker to determine the jumboization investment. It is in line with the real practices followed by the U.S. Navy. In other words, if one examines the real examples of jumboization in U.S. Navy history, he/she observes that demand for fuel by the receiving ships has been an influential factor to decide on jumboizing the replenishment oilers.

GBM part of this assumption needs statistical validation. Unfortunately, we lack data showing the demand amount transported by a particular replenishment oiler. Instead, we encounter annually published U.S. Navy reports (Shannon 2014 and other similar reports published in previous years) depicting the total amount of fuel transported by all replenishment oilers in a year. Therefore, we conduct statistical tests on this data set (see Appendix 3.A) by assuming that it is representative of data set of fuel amount transported by a single replenishment oiler. These tests reveal that GBM assumption is valid.

There exist several studies in the literature assuming demand as uncertain parameter following GBM process. For instance, demand as number of passengers per year and per month are used by Pereira et al. (2006) and Marathe and Ryan (2005), respectively, in airline context.

Assumption 2: We consider only one replenishment oiler to evaluate the jumboization operation conducted on it.

The U.S. Navy has currently six fleets serving at the world seas. The complete list of these fleets can be given as follows (see, e.g., Wikipedia 2018a): 3rd fleet serves in eastern and northern Pacific Ocean; 4th fleet serves in Central and South America; 5th fleet serves in Persian Gulf, Red Sea and Arabian sea; 6th fleet serves in Europe and Africa; 7th fleet serves in western Pacific Ocean. Finally, 10th fleet serves as a leading role in cyber warfare program of the U.S. Navy, which does not have a specific location (Wikipedia 2018c).

Each fleet can be thought as an individual unit as they are separate and they have different missions and commanders. To the best of our knowledge, each replenishment oiler has been assigned to a specific fleet to transport the fuel and other supporting items to the ships in the fleet. Due to this separated property of the U.S. Navy, we focus on one of the fleets. Moreover, one can see that fleet regions can be separated into sub-regions with respect to port locations. Therefore, it is possible to take into account only one of these sub-regions and it can be assumed that only one replenishment oiler departs from a specified port. In this case, the problem turns into a smaller problem, which focus on only one replenishment oiler. Another supporting fact is that the reports published by The U.S. Navy's Military Sealift Command do not reveal any real example of the situation that multiple replenishment oilers depart from a location and replenish the receiving ships at the same time (Shannon 2014 and other reports published by Military Sealift Command in different years). It implies that replenishment oilers operate individually at seas.

In line with this assumption, Blackman (2012) creates a sub-region around Monterey port of California to run his model and he assumes that only one replenishment oiler departs from the port. Furthermore, there exist several studies in ship scheduling literature, which take into account only one ship. For instance, Besbes and Savin (2009) deal with single-vessel (belongs to either liner or tamper type) profit maximization problem under fuel cost uncertainty. Kim et al. (2012) minimize overall cost of a single ship related to bunkering decisions.

Besides the replenishment oiler, our framework allows to consider multiple receiving ships under the condition that they are approximately at the same location while being replenished. Historical data of replenishment locations (Blackman 2012) supports that multiple receiving ships can be replenished within a very small region at sea. For instance, the receiving ships around Monterey port of California were replenished more than 100 times within 50 square miles over a couple of weeks. Moreover, it is a fact that a replenishment oiler can often replenish two receiving ships simultaneously (see, e.g., Marconi 2012).

Assumption 3: The port of the replenishment oiler and the location of the receiving ships do not change. In other words, the replenishment oiler makes round trip between two specified locations. The distance between the port and the location is denoted by X in nautical miles (1 nautical mile is equal to 1852 m).

Note that the replenishment oiler travels in distance X twice, while transporting the fuel to the receiving ships and returning to its port.

As stated in the explanation of Assumption 2, it is a fact that replenishment of the receiving ships can happen within a very small area. Taking into account this fact, Blackman (2012) simulates and predicts future replenishment locations in eastern and northern Pacific. His simulation results show that replenishment locations change by less than 20 nautical miles.

There exist several studies, which make a similar assumption in ship scheduling literature. For instance, Boros et al. (2008) take into account two shipping companies with different objectives as sides of a supply chain contract. The authors determine optimal cycle time of the vessels by assuming that the vessels operate between two specified ports. Another study conducted by Chen et al. (2007) show the solvability of special cases of bi-directional vessel routing as a linear program. They assume that ships operate between two specified locations (see also Lei et al. 2008; Koenigsberg and Lam 1976).

Assumption 4: The replenishment oiler moves at a constant speed, denoted by S in knots (1 knot is equal to 0.514 m/sn), in each round trip. It means that it moves with a fixed speed in transporting the fuel to the receiving ships and in returning to the port and this speed remains constant in the next round trips.

This assumption can be justified in two different aspects. First, the speed change of the replenishment oiler may have a dynamic aspect in one-way trip, but we simplify it by saying that there exists an average speed, calculated over one-way trip. Accepting average speed rather than dealing with dynamic nature of speed is a common trend in the literature. For instance, Raff (1983) says that travel distances divided by an average speed gives acceptable travel times for private sector ship transportation. Aydin et al. (2017) assume in their model that the speed of a ship does not change in a trip from one port to the next port and it is called as average speed (see also Ball et al. 1983; Besbes and Savin 2009; Kim et al. 2012).

Second, our assumption implies that average speed remains constant through multiple round trips. This can be rationalized as follows: By Assumption 3, travel distances of the replenishment oiler do not change over the time horizon and by Assumption 1, we state calls for demand occur in each equal time periods. Furthermore, in reality, the engines of the replenishment oilers are installed with a maximum capacity to be able to carry maximum loads of the ships. Therefore, no matter how much load it carries, the replenishment oiler is able to keep the constant speed. Since it travels the same distance multiple times throughout the modeling horizon, the decision maker can choose an appropriate speed for their operational purposes. In line with this justification, Ronen (2011) is able to derive the optimal average speed for a fixed fleet by taking into account weekly demand occurrences in the ports, which facilitates the derivation procedure. Moreover, Fagerholt (1999) determines optimal fleet size and optimal route for each selected ship to transport cargos from a central depot to multiple off-shore locations. Main assumption of his study is that all the ships selected have a common speed and it does not change over time, which is claimed to be a case in many of the practical problems. He also emphasizes that the model does not deal with temporal aspect of the problem as the model does not try to schedule all the ships by considering time windows (see also Hemmati et al. 2014 and Christiansen et al. 2007).

Besides the above explanation, we note that dynamic aspect of speed change may not be incorporated into our mathematical model as it is not obvious to observe how the speed changes by time (e.g. undefined mathematical formulation).

Note that $(1852X/0.514S)/3600 \approx X/S$ gives the number of hours needed for the replenishment oiler to transport the fuel to the receiving ships. Therefore, unit time duration should be larger than or equal to $2X/S$. If it is strictly larger than $2X/S$, it shows that the replenishment oiler completes its task and it stays at the port without functioning until another call for replenishment.

For further discussions about Assumptions 3 and 4, interested readers can see our Discussion section.

Assumption 5: Jumboization is the only option to be considered. Other managerial options such as mothballing, incremental increase of capacity, abandoning and purchasing of the replenishment oiler are not considered in this chapter.

This assumption can be relaxed in a couple of ways. First, decommissioning of the replenishment oiler can be taken into consideration along with jumboization option, although we do not know in advance if such a model can be solvable in closed form. On the other hand, several numerical techniques proposed in the literature can be used to derive solutions. Second, purchasing of the replenishment oiler and its time might be turned into a managerial decision unlike its compulsory situation in this chapter. However, we do not consider these relaxations in this chapter because our primary focus is jumboization option and its expected time.

Assumption 6: The replenishment oiler is non-depreciating and thus, jumboization is an infinitely lived option.

Although this assumption seems to be impractical, we require it because analytical framework results in closed-form solutions only if this assumption is made (Dixit and Pindyck 1994). In this chapter, we present a discrete counterpart model of our framework to verify and validate our closed-form solutions.

Assumption 7: Let I_{flex} and I_{fixed} be the costs incurred during jumboization operations for flexible and fixed designs, respectively. It is assumed that $I_{flex} < I_{fixed}$.

This is intuitively true because the decision maker pays less for jumboization due to the fact that flexible design is already prepared for jumboization. Otherwise, flexible design would not have any competitive advantage, if we especially consider an additional upfront cost, which is incurred at the initial stage of ship building to have flexible design.

Upfront cost for flexible design can arise from stronger hull structure by more advanced scantlings and this cost is denoted as I_0 in this study. Buxton and Stephenson (2001) state that the hull of jumboized ship needs to have additional strengthening because it is subject to higher bending moments and shear forces. Bending moment is defined as the amount of bending applied to the hull by the external forces, measured in ton-meters (see, e.g., Bulk Carrier Guide 2010). It is basically caused by two different forces; weight on the hull (acting downwards) and buoyancy (acting upwards). If the weight distribution is higher than buoyancy in the mid-section of the hull, bending moment is called sagging. On the other hand, if the weight distribution is higher than buoyancy in the stern (backward part of the hull) and bow (forward part of the hull) sections, it is named as hogging. Besides weight and buoyancy, forces caused by waves can also result in bending moments (see, e.g., Marine Survey Practice 2013). As for shear force (measured in tons), it is defined as the force applied at any point of the length of the ship, which tends to move one part of the hull to adjacent position vertically (see, e.g., Marine Survey Practice 2013). In other words, it is the tendency of breaking apart of the hull. It is basically caused by uneven load distribution and unbalanced vertical forces. Literature of ship design suggests to use higher strength steels to reduce bending moments and shear forces. If the decision maker decides using higher strength steels at initial design by paying upfront cost I_0, the effect of higher bending moments and shear forces, resulted from jumboization, can be balanced.

In the subsequent subsections, we first introduce the benefit gained through jumboization, which serves as the objective maximized in our model. We then present the way of determining the value of jumboization option as well as its expected time by means of an analytical framework and a discrete model.

Fuel Cost Saving Gained Through Jumboization

In addition to capacity increase for cargo, the literature of mechanical design of the ships reveals that lengthening of a ship generally decreases the wave-making resistance of the ship (see, e.g., ABS 2017). Since resistance against the ship is directly proportional to fuel consumption amount (Ericson and Lake 2014), we state that jumboization generally leads to fuel cost saving. Note that we have addressed two different fuel types so far. Demand refers to cargo fuel, which is transported by the replenishment oiler to the receiving ships. Bunker fuel refers to the fuel, which is consumed to propel the replenishment oiler. To better reflect this difference, we use tons as the unit of cargo fuel and gallon as the unit of bunker fuel.

Sen and Yang (1998) indicate that power (required to propel the replenishment oiler) and fuel consumption is proportional. In literature, there are several expressions for power, which approximate the real power required by a ship. In this study, we present the most elaborative and the most precise approximation. Table 3.1 shows notations and corresponding definitions of basic design parameters of a ship. Other parameters used throughout the study and their definitions are given in the text.

Table 3.1 *Notations associated with ship design and their definitions*

Notations	Definition	Explanation
$\mathcal{L}$	Light ship mass	Mass of the ship's hull and other permanent items in the ship (tons)
Δ	Displacement	Light ship mass plus the maximum amount of cargo that the ship can carry (tons). It means displacement refers to maximum tons that the ship can carry (see, e.g., Archives 2018).
P	Power	The maximum power required to propel the replenishment oiler (kW)
L	Length	Length of the replenishment oiler (m)
$\mathcal{B}$	Breadth	Width of the replenishment oiler (m)
$\mathbb{D}$	Draft	Vertical distance between the waterline and the bottom of the hull (m)
$\mathcal{D}$	Depth	Vertical distance between the top and the bottom of the hull (m)

One prominent approach to approximate the required power is called Admiralty method, which is an equation including Admiralty coefficient (Schneekluth and Bertram 1998). Admiralty coefficient is a constant for similar ships (Similar ships are those that have similar design parameters such as speed, length and mass). It is estimated for a newly designed ship by analyzing the parent ships' data, which have very similar properties in aspects mentioned above. Admiralty coefficient gives the approximate relations between the ship's speed, displacement and required power and this relation is stated as

$$P = \frac{\Delta^{2/3} S^3}{\mathcal{A}} \tag{3.2}$$

where $\mathcal{A}$ is Admiralty coefficient (see, e.g., Man 2011). For example, higher $\mathcal{A}$ means less power is required for a newly designed ship. Equation (3.2) derives from Bernoulli law and resistance against the ship (for derivation details, see Appendix 3.B). Schneekluth and Bertram (1998) give

$\mathcal{A}$ with a unit of ton$^{2/3}$knot^3/kW. Note that power expressed in Equation (3.2) represents the maximum power (it is often called installed power) required to propel the ship because Δ, by definition, is the maximum tons that a replenishment oiler can carry.

Significant studies have been conducted so far to find more precise variants of Equation (3.2). Sen and Yang (1998) accomplish by defining a relation between $\mathcal{A}$ and Froude number, denoted by $\mathbb{F}$. Froude number is an important figure used to calculate the wave-making resistance of a partially submerged body. It is given as

$$\mathbb{F} = \frac{0.514S}{\sqrt{gL}} \tag{3.3}$$

where g is gravitational constant (m/sn^2). Higher Froude number means that the partially submerged object has higher wave-making resistance. It is discovered by Sen and Yang (1998) that the relation between $\mathcal{A}$ and $\mathbb{F}$ is linear. Thus, they write that

$$\mathcal{A} = m + n\frac{0.514S}{\sqrt{gL}} \tag{3.4}$$

where $m > 0$ and $n < 0$ are coefficients. When Equation (3.4) is plugged into Equation (3.2), it gives

$$P = \frac{\Delta^{2/3}S^3}{m + n\frac{0.514S}{\sqrt{gL}}} \tag{3.5}$$

under the constraints $L/\mathcal{B} \geq 6$, $L/\mathcal{D} \leq 15$ and $L/\mathbb{D} \leq 19$. These constraints stem from the mechanical principles. For instance, increasing the length causes higher chances to roll down. In addition to mechanical constraints, the topological barriers of routes require the ships not to excess some levels in these dimensions. For instance, in the case of Panama Canal, the ships must have length less than 289 meters. In Canal St. Lorenz, this constraint turns out to be much tighter and the length should have less than 222 meters (Papanikolaou 2014).

Equation (3.5) captures many of the realities. For example, at constant displacement, if length increases, the maximum power to propel the ship decreases. It supports the fact that longer hull creates less resistance and leads to less power requirement.

Sen and Yang (1998) give the expressions for m and n, as well. Their analysis results in

$$m = 4977B^2 - 8105B + 4456 \tag{3.6}$$

$$n = -10847B^2 + 12817B - 6960 \tag{3.7}$$

where B is block coefficient, which can be defined as follows: Imagine that a rectangular prism is built around the submerged part of the ship. The proportion of the real volume of this part to the volume of rectangular prism is defined as block coefficient (see, e.g., Man 2011). Block coefficient is said to increase as a result of jumboization (Ericson and Lake 2014).

Sen and Yang (1998) state that the maximum daily consumption of bunker fuel is a linear function of P, i.e., $0.0046P + 0.2$. Thus, the maximum amount of bunker fuel consumption in a one-way voyage can be written as $[(0.0046P + 0.2)/24](X/S)$. We note that the result of this calculation is fuel consumption in tons (see, e.g., Sen and Yang 1998). Therefore, there needs to be a conversion from tons to gallon by using density value of bunker fuel. In mathematical framework, we omit this conversion, but we show it in the numerical example.

Amount of bunker fuel consumed per unit displacement (gallon/ton) in one-way voyage is written as

$$\mathcal{F} = \frac{(0.0046P + 0.2)X}{24S\Delta} \tag{3.8}$$

Since Δ includes both $\mathcal{L}$ and D (note that we drop the subscript t from D_t because it is irrelevant in this discussion), separation of round trip voyages of the replenishment oiler turns out to be important. While it carries $\mathcal{L}$ and D to the receiving ships in one direction, it carries only $\mathcal{L}$ while returning to the port. Hence, the fuel cost ($/unit time) is given as

$$\mathcal{F}C(\mathcal{L} + D) + \mathcal{F}C\mathcal{L} \tag{3.9}$$

where C is cost of unit bunker fuel ($/gallon). Since jumboization changes Δ, $\mathcal{L}$, L and B; $\mathcal{F}$ and $\mathcal{L}$ expressions in Equation (3.9) vary from pre-jumboization case to post-jumboization case. Let $\mathcal{F}_1$ and $\mathcal{F}_2$ have the same definitions as $\mathcal{F}$, but denote pre-jumboization and post-jumboization cases, respectively (Make the same definitions for $\mathcal{L}$ as well). Note that since $\mathcal{F}$ is a function of Δ, L and B, these parameters have also subscripts 1 and 2 to denote pre-jumboization and post jumboization cases, respectively. Therefore, fuel saving per unit time due to jumboization can be expressed as

$$[\mathcal{F}_1C(\mathcal{L}_1 + D) - \mathcal{F}_2C(\mathcal{L}_2 + D)] + [\mathcal{F}_1C\mathcal{L}_1 - \mathcal{F}_2C\mathcal{L}_2] \tag{3.10}$$

which is simplified as

$$2(\mathcal{F}_1\mathcal{L}_1 - \mathcal{F}_2\mathcal{L}_2)C + (\mathcal{F}_1 - \mathcal{F}_2)CD \tag{3.11}$$

Note that the first part of expression (3.11) might be negative. On the other hand, $\mathcal{F}_1 - \mathcal{F}_2$ should be positive so that whole expression can be positive for some large D. It emphasizes that there is a level for D, above which the expression is positive and jumboization is effective in bringing about the fuel cost saving.

Option Valuation for Jumboization in Analytical Framework

Since jumboization is an option for the decision maker, it is exercised when financial benefits from jumboization start being justified. Hence, this problem can be treated as optimal stopping problem. In other words, there exists a D^* (threshold demand level), above which the decision maker decides on jumboization and below which, he/she does not prefer jumboization. When the replenishment oiler is jumboized, the decision maker starts gaining all future fuel savings right after the jumboization. Assuming that jumboization is done at the level of D_x (note that x does not denote time, instead D_x is just a notation used to denote demand level at which jumboization is done), the value of project (project in this context is the jumboized replenishment oiler) is expressed as

$$V(D_x) = E\left[\int_0^{\infty} [2(\mathcal{F}_1\mathcal{L}_1 - \mathcal{F}_2\mathcal{L}_2)C + (\mathcal{F}_1 - \mathcal{F}_2)CD_t]e^{-\rho t}dt\right] \tag{3.12}$$

where ρ (%/unit time) is risk-adjusted discount rate and it is exogenously specified. Note that lower bound of integral is accepted as 0, and it corresponds the demand level denoted by D_x. It is assumed in real options context that $\rho > \alpha$ because otherwise, waiting longer for the investment always becomes better policy (Dixit and Pindyck 1994). Equation (3.12) can be simplified as

$$V(D_x) = \frac{2(\mathcal{F}_1\mathcal{L}_1 - \mathcal{F}_2\mathcal{L}_2)C}{\rho} + (\mathcal{F}_1 - \mathcal{F}_2)CE\left[\int_0^{\infty} D_t e^{-\rho t}dt\right] \tag{3.13}$$

In order to calculate the integration in Equation (3.13), we need to change the order of integration and expectation. Some conditions should hold so as to change the order according to Fubini's theorem (Klebaner 2005). Interested readers can review Appendices 3.C and 3.D to figure out what the theorem is and how it works in our case. As a result, changing order of integration and expectation is viable and the solution is derived as

$$V(D_x) = \frac{2(\mathcal{F}_1\mathcal{L}_1 - \mathcal{F}_2\mathcal{L}_2)C}{\rho} + \frac{(\mathcal{F}_1 - \mathcal{F}_2)C}{\rho - \alpha} D_x \tag{3.14}$$

Equation (3.14) can be interpreted as annual perpetuity. Since $2(\mathcal{F}_1\mathcal{L}_1 - \mathcal{F}_2\mathcal{L}_2)C$ does not grow by time, it is discounted with ρ. On the other hand, $(\mathcal{F}_1 - \mathcal{F}_2)CD$ in expression (3.11) grows with the rate of α and discounted with the rate of ρ. Therefore, the net discount rate turns out to be $\rho - \alpha$ (Dixit and Pindyck 1994).

The value of the option to jumboize the replenishment oiler, denoted by F (note that $\mathcal{F}$ is amount of bunker fuel consumed per unit displacement in one-way voyage, F denotes the value of option to jumboize, and $\mathbb{F}$ is Froude number), has a value. It evolves as

$$\rho F dt = E[dF] \tag{3.15}$$

which means that the option gains capital appreciation before jumboization. It does not have a term related to fuel saving because fuel saving appears after jumboization. Since F is a function of D, one can derive the explicit form of dF by applying Ito's lemma. That is,

$$dF = \left[\alpha DF' + \frac{1}{2}\sigma^2 D^2 F''\right] dt + \sigma DF' dz \tag{3.16}$$

and

$$E[dF] = \left[\alpha DF' + \frac{1}{2}\sigma^2 D^2 F''\right] dt \tag{3.17}$$

If Equation (3.17) is plugged into Equation (3.15) and dt terms cancel each other, one obtains

$$\frac{1}{2}\sigma^2 D^2 F'' + \alpha D F' - \rho F = 0 \tag{3.18}$$

Second-order homogenous differential equation has a general solution of $F(D) = AD^{\beta}$. It can be written as a precise expression as $F(D) = A_1 D^{\beta_1} + A_2 D^{\beta_2}$ where $\beta_1 > 1$ and $\beta_2 < 0$ (see Appendix 3.E). In order to solve this equation, we need boundary conditions. One boundary condition is $\lim_{D \to 0} F(D) = 0$. It is intuitively true because when demand level approaches to 0, the option to jumboize the replenishment oiler becomes ineffective. It results in $F(D) = A_1 D^{\beta_1}$. Other boundary conditions can be written for threshold demand value. At D^*, one can write that

$$F(D^*) = V(D^*) - I \tag{3.19}$$

$$F'(D^*) = V'(D^*) \tag{3.20}$$

where I (can be either I_{fixed} or I_{flex}) is the investment cost incurred during jumboization operations. Equation (3.19) is called as value-matching condition and it means that the decision maker gets benefits from jumboization via fuel saving in exchange of jumboization cost. Equation (3.20) is called smooth-pasting condition and it guarantees optimality at D^*. With these conditions, D^* and $F(D)$ can be obtained as (see Appendix 3.F)

$$D^* = \left(I - \frac{2(\mathcal{F}_1\mathcal{L}_1 - \mathcal{F}_2\mathcal{L}_2)C}{\rho}\right)\frac{\beta_1(\rho - \alpha)}{(\beta_1 - 1)(\mathcal{F}_1 - \mathcal{F}_2)C} \tag{3.21}$$

$$F(D) = \left[\frac{(\mathcal{F}_1 - \mathcal{F}_2)C}{(\rho - \alpha)\beta_1}D\right]^{\beta_1}\left[\left(I - \frac{2(\mathcal{F}_1\mathcal{L}_1 - \mathcal{F}_2\mathcal{L}_2)C}{\rho}\right)\frac{1}{\beta_1 - 1}\right]^{1-\beta_1} \tag{3.22}$$

It is noted that $I - \frac{2(\mathcal{F}_1\mathcal{L}_1 - \mathcal{F}_2\mathcal{L}_2)C}{\rho} > 0$ should hold for obtaining $D^* > 0$. As will be seen in numerical example, design parameter values of a real replenishment oiler satisfy $\mathcal{F}_1\mathcal{L}_1 - \mathcal{F}_2\mathcal{L}_2 < 0$, which does not cause any problem in this respect. However, if numerical values cause $\mathcal{F}_1\mathcal{L}_1 - \mathcal{F}_2\mathcal{L}_2 > 0$ and if this results in $D^* < 0$, we need to enforce $I - \frac{2(\mathcal{F}_1\mathcal{L}_1 - \mathcal{F}_2\mathcal{L}_2)C}{\rho} > 0$ by adjusting numerical values.

Given that D^* has the form in Equation (3.21), expected time for demand process to pass from an arbitrary D_0 (demand value at time 0) to D^* (under the condition that $D_0 < D^*$ because D has positive drift) is given by

$$\tau = \frac{\ln D^* - \ln D_0}{\alpha - \sigma^2/2} \tag{3.23}$$

Note that we need to assume $\alpha - \sigma^2/2 > 0$ for τ to be positive (see, e.g., Min et al. 2012).

Discrete Counterpart of Continuous Model

Closed-form solution of D^* emerges as a result of Assumption 6, which states that option life for jumboization and service life of the replenishment oiler are infinite. A question might arise as to how reliable this solution is because the model with Assumption 6 deviates from reality. In this respect, we think that it might be beneficial and illuminating to create a discrete model in order to show that the solution resulting from the discrete model is close enough to the solution resulting from the analytical model.

Discretization of uncertain parameter

Several discrete approaches have been proposed so far to solve the real options problem. The binomial lattice approach, which is firstly developed by Cox et al. (1979), has become one of the prominent methods in this area. The basic idea of the binomial lattice is to approximate GBM process with up and down movements with corresponding probabilities. It is proven that if up movement factor (u), down movement factor (d) and the movement probabilities are chosen as in Equations (3.24) - (3.26) (p for up movement and $1-p$ for down movement), the binomial lattice approximates GBM process well:

$$u = e^{\sigma\sqrt{\Delta t}} \tag{3.24}$$

$$d = e^{-\sigma\sqrt{\Delta t}} \tag{3.25}$$

$$p = \frac{1}{2} + \frac{1}{2}\left(\frac{\alpha - \sigma^2/2}{\sigma}\right)\sqrt{\Delta t} \tag{3.26}$$

To clarify, D_0 can take two values at the next time point; either uD_0 with probability p or dD_0 with probability $1-p$. Note that this lattice is called recombining lattice because after two time points, D_0 appears again as $u \cdot d = 1$. In demand lattice, we denote demand values with two subscripts; t for time points and k for states. $D_{(t,k)}$ denotes demand value at time point t and state k. Time point t represents the end of time period t and state is just numbering of the nodes of the lattice starting from 1 at the uppermost node and incrementing by 1 through the bottommost node of the lattice for each t. Note that in Equations (3.24) - (3.26), Δt denotes the length of one time period (as a fraction or a multiple of the length of unit time, which is specified in analytical framework) in the binomial lattice. Δt can vary from a few days to several years. Note also that since the unit of σ is %/unit time, Δt is equal to 1 if the length of Δt is set equal to length of unit time.

Real options evaluation requires to have risk-neutral probabilities of up and down movements instead of p given in Equation (3.26). Risk-neutral probability of up movement is given as

$$q = \frac{1 + r\Delta t - d}{u - d} \tag{3.27}$$

with the condition that $d < 1 + r\Delta t < u$. Note that r (%/unit time) is the risk-free interest rate (it is generally stated that $\rho - r > 0$ should hold because risk-adjusted discount rate involves a positive risk premium) and we multiply it with Δt to find the accurate interest rate in Δt.

Option valuation for jumboization in discrete model

There are three steps in option valuation for jumboization. The first step, as already described above, is the creation of evolution of demand process. Having set a terminating time point, denoted as T (years or a fraction of one year) and set Δt, number of time periods (found by $T/\Delta t$) and corresponding labeling of time points (starting from 0, which denotes the current time, and goes through $T_{\Delta t} = T/\Delta t$, which denotes the last time point) are determined. For instance, if $T = 10$ years and $\Delta t = 0.5$ years are chosen, the number of periods turns out to be 20 and time points start from 0 and goes through 20. In this case, $D_{(17,1)}$ represents the demand value at time point 17 (at the end of 8.5 years) and state 1, which is the highest demand value for $t = 17$ on this lattice.

The second step is the creation of the lattice, which represents the evolution of the value of the replenishment oiler in the case that it has already been jumboized at time 0. In other words, fuel saving benefit is in place for each node of the lattice. Valuation proceeds in a backward manner. That is, values should be assigned first for all the nodes at time point $T_{\Delta t}$. At the end of modeling horizon, we assume that there is neither cost, nor a salvage value in order to keep consistency with the analytical model. Therefore, value 0 is assigned for all the nodes at time point $T_{\Delta t}$. In mathematical terms, we denote it as $\mathcal{V}_{(T_{\Delta t},k)} = 0, \forall k \in [1, T_{\Delta t} + 1]$ where $\mathcal{V}_{(t,k)}$ denotes the value of the replenishment oiler at time point t and state k in the case that jumboization is in place.

Having assigned $\mathcal{V}_{(T_{\Delta t},k)} = 0$, we go one time point back and determine the value for all the nodes at time point $T_{\Delta t} - 1$. For all k, we calculate

$$\mathcal{V}_{(T_{\Delta t}-1,k)} = \frac{\left[2(\mathcal{F}_1\mathcal{L}_1 - \mathcal{F}_2\mathcal{L}_2)C + (\mathcal{F}_1 - \mathcal{F}_2)CD_{(T_{\Delta t}-1,k)}\right]}{1 + r\Delta t} \tag{3.28}$$

We assume that all cash flow occurs at the end of Δt in accordance with the traditional approach in engineering economist. Since node values at time point $T_{\Delta t}$ are all 0, we do not include risk-neutral expected value of the subsequent nodes in Equation (3.28). For an arbitrary $t < T_{\Delta t} - 1$, we make the same calculation as Equation (3.28) except we also include risk-neutral expected value of the subsequent nodes. In other words,

$$\mathcal{V}_{(t,k)} = \frac{\left[2(\mathcal{F}_1\mathcal{L}_1 - \mathcal{F}_2\mathcal{L}_2)C + (\mathcal{F}_1 - \mathcal{F}_2)CD_{(t,k)}\right]}{1 + r\Delta t} + \frac{\mathcal{V}_{(t+1,k)}q + \mathcal{V}_{(t+1,k+1)}(1-q)}{1 + r\Delta t} \tag{3.29}$$

The third step is to create a lattice, which shows the evolution of value of the replenishment oiler with jumboization option. In this lattice, the decision maker chooses either jumboizing the replenishment oiler or continuing with the non-jumboized situation. We start the procedure by assigning value 0 for all the nodes at time point $T_{\Delta t}$. Since this is the expiration date of jumboization option and the end of service life of the replenishment oiler, the decision maker does not choose making investment because there is not any future benefit. Mathematically, it is stated as $\hat{\mathcal{V}}_{(T_{\Delta t},k)} = 0, \forall k \in [1, T_{\Delta t} + 1]$ where $\hat{\mathcal{V}}_{(t,k)}$ denotes the value of the replenishment oiler at time point t and state k with jumboization option. For the time point $T_{\Delta t} - 1$,

$$\hat{\mathcal{V}}_{(T_{\Delta t}-1,k)} = \max\left\{\mathcal{V}_{(T_{\Delta t}-1,k)} - I; 0\right\} \tag{3.30}$$

Note that if $\mathcal{V}_{(T_{\Delta t}-1,k)} - I > 0$, then the decision maker invests. Otherwise, the replenishment oiler continues being in service without jumboization. The first part of the maximization of Equation (3.30) is interpreted as the immediate benefit from jumboization. The second part of it is called the continuation value and it is zero for $T_{\Delta t} - 1$ because subsequent nodes at time $T_{\Delta t}$ have all value 0. For the time points $t < T_{\Delta t} - 1$, we calculate

$$\hat{\mathcal{V}}_{(t,k)} = \max\left\{\mathcal{V}_{(t,k)} - I; \frac{\hat{\mathcal{V}}_{(t+1,k)}q + \hat{\mathcal{V}}_{(t+1,k+1)}(1-q)}{1 + r\Delta t}\right\} \tag{3.31}$$

and determine if the decision maker invests. The continuation value is now expressed as the risk-neutral expected value of the subsequent nodes and discounted one period back.

Determining threshold demand values in discrete model

Our purpose in creating discrete model is to compare D^* value of analytical model with $D^*(t)$ values of the binomial model. Note that there is not a single D^* value in the binomial model; instead, it changes by time. The reason is that the decision maker jumboizes the replenishment oiler at higher values of demand when the time approaches to the end of service life of the replenishment oiler. Therefore, it indicates that $D^*(t)$ is an increasing curve. The following list elaborates the way of calculating $D^*(t)$ (Ashuri et al. 2011):

(i) Let $t = 0$. Since threshold demand level is the level at which the decision maker is indifferent between making the investment or continuing with non-investment situation, we seek for

$$\mathcal{V}_{(0,1)} - I \cong \frac{\hat{\mathcal{V}}_{(1,1)}q + \hat{\mathcal{V}}_{(1,2)}(1-q)}{1 + r\Delta t} \tag{3.32}$$

where left-hand side is the immediate benefit from the investment and right-hand size is the continuation value. To clarify, we solve the lattice model with three steps defined previously by changing D_0 until we observe Equation (3.32) holds. D_0 which satisfies the approximate equality stated in Equation (3.32) is determined as $D^*(0)$. As an initial guess, D_0 value, which is used for evaluation of jumboization option in the preceding section can be adopted again. If left-hand side of Equation (3.32) is larger than the right-hand side, then it is an indication for $D^*(0) < D_0$. In this case, we decrease D_0 and solve three steps again. If right-hand side of Equation (3.32) is larger than the left-hand side, then $D^*(0) > D_0$. Thus, we increase D_0 and solve three steps.

(ii) Increment t by 1 and create a partial demand lattice with one initial node at time point t and remaining nodes through time point $T_{\Delta t}$. Having created demand lattice, we repeat the above procedure defined in (i) and find $D^*(t)$. After finding $D^*(t)$ for t, we increment again t by 1 and repeat this procedure. We terminate it once we find $D^*(T_{\Delta t} - 1)$.

In the following section, we present the results of sensitivity analysis conducted on D^*, with respect to relevant parameters, to derive significant policy insights for the decision maker.

Sensitivity Analysis and Managerial Insights

The following propositions list the results of analysis by taking into account the most significant parameters:

Proposition 1: $\frac{\partial D^*}{\partial \mathcal{L}_1} < 0$ and $\frac{\partial D^*}{\partial \mathcal{L}_2} > 0$

It is straightforward to see these results from Equation (3.21). If $\mathcal{L}_1$ is larger, D^* decreases because the decision maker tends to gain more fuel saving and jumboizes the replenishment oiler earlier because of the fact that larger mass leads to more fuel cost. On the other hand, if $\mathcal{L}_2$ is larger, then the decision maker waits for higher demand values to jumboize because larger mass after jumboization has less impact on fuel saving.

Proposition 2: $\frac{\partial D^*}{\partial L_1} > 0$ and $\frac{\partial D^*}{\partial L_2} < 0$

Since D^* depends on L_1 and L_2 via $\mathcal{F}_1$ and $\mathcal{F}_2$, respectively; we reach the conclusion with chain rule. It can be obtained that $\frac{\partial D^*}{\partial \mathcal{F}_1} < 0$, $\frac{\partial D^*}{\partial \mathcal{F}_2} > 0$, $\frac{\partial \mathcal{F}_1}{\partial L_1} < 0$ and $\frac{\partial \mathcal{F}_2}{\partial L_2} < 0$. Therefore, $\frac{\partial D^*}{\partial L_1} = \frac{\partial D^*}{\partial \mathcal{F}_1}\frac{\partial \mathcal{F}_1}{\partial L_1} > 0$ and $\frac{\partial D^*}{\partial L_2} = \frac{\partial D^*}{\partial \mathcal{F}_2}\frac{\partial \mathcal{F}_2}{\partial L_2} < 0$. They indicate that the decision maker tends to jumboize the replenishment oiler later when its initial length is larger. The reason is that longer hull already provides fuel efficiency. On the other hand, the decision maker would like to jumboize the replenishment oiler earlier if its length after jumboization is larger because more fuel saving, which arises from longer hull structure, are expected to be adopted.

Proposition 3: $\frac{\partial D^*}{\partial \Delta_1} > 0$ and $\frac{\partial D^*}{\partial \Delta_2} < 0$

D^* depends on Δ_1 and Δ_2 via $\mathcal{F}_1$ and $\mathcal{F}_2$, respectively. We can see that $\frac{\partial \mathcal{F}_1}{\partial \Delta_1} < 0$ and $\frac{\partial \mathcal{F}_2}{\partial \Delta_2} < 0$. Therefore, $\frac{\partial D^*}{\partial \Delta_1} = \frac{\partial D^*}{\partial \mathcal{F}_1}\frac{\partial \mathcal{F}_1}{\partial \Delta_1} > 0$ and $\frac{\partial D^*}{\partial \Delta_2} = \frac{\partial D^*}{\partial \mathcal{F}_2}\frac{\partial \mathcal{F}_2}{\partial \Delta_2} < 0$. These results show that the decision maker tends to jumboize the replenishment oiler later if its initial displacement is larger. The reason is that larger initial displacement results in less fuel consumption per ton displacement and thus jumboization does not seem to be immediate requirement. On the other hand, larger displacement after jumboization generates less fuel consumption per ton displacement and the decision maker tends to capitalize on it earlier.

Proposition 4: $\frac{\partial D^*}{\partial C} < 0$ and $\frac{\partial D^*}{\partial I} > 0$

These results can be derived from D^* expression, Equation (3.21). If unit fuel cost increases, the decision maker tends to jumboize the replenishment oiler earlier because he/she avoids being exposed to more fuel cost and makes use of jumboization. If jumboization cost increases, then investment is delayed because the decision maker expects to observe higher demand values and to gain more fuel saving to compensate higher investment cost.

Proposition 5: $\frac{\partial D^*}{\partial \sigma} > 0$

In order to determine the sensitivity of D^* with respect to σ, we need to investigate the sensitivity of β_1 with respect to σ. It can be verified that $\frac{\partial \beta_1}{\partial \sigma} < 0$ (see Appendix 3.G) and $\frac{\partial D^*}{\partial \beta_1} < 0$ (see Appendix 3.H). Thus, $\frac{\partial D^*}{\partial \sigma} = \frac{\partial D^*}{\partial \beta_1}\frac{\partial \beta_1}{\partial \sigma} > 0$. It indicates that when volatility of uncertainty increases, the decision maker tends to avoid making critical decisions which incur huge costs, and thus it causes delaying the jumboization operations.

Proposition 6: $\frac{\partial D^*}{\partial X} < 0$

Interested readers can review Appendix 3.I for derivation details. This result indicates that if the replenishment oiler becomes more active, then the decision maker tends to jumboize it earlier because fuel saving benefit appears more in longer distances.

In the next section, we provide a managerial guideline concerning the choice between flexible and fixed design, and we propose conditions under which flexible design becomes financially superior over fixed design.

Choice Between Flexible and Fixed Designs

The U.S. Navy does not necessarily need to adopt flexible design. Thus, a question arises as to under what condition flexible design is more preferable than fixed design. Upfront cost incurred for flexible design (I_0) represents a critical part of the answer to this question.

As the history of jumboization shows, a replenishment oiler with fixed design can also be jumboized. Thus, the replenishment oiler with fixed design has also option value, contingent upon the demand uncertainty. To compare flexible and fixed designs, option values at time 0 of both

($F_{flex}(D_0)$ and $F_{fixed}(D_0)$ are option values of flexible and fixed designs, respectively, at time 0) should be taken into account. Flexible design should be preferred over fixed design in the case that the difference between $F_{flex}(D_0)$ and $F_{fixed}(D_0)$ is larger than upfront cost. In other words, flexible design should be preferred if

$$I_0 < F_{flex}(D_0) - F_{fixed}(D_0) \tag{3.33}$$

or,

$$\begin{aligned} I_0 < &\left[\frac{(\mathcal{F}_1 - \mathcal{F}_2)C}{(\rho - \alpha)\beta_1} D_0\right]^{\beta_1} \left[\left(I_{flex} - \frac{2(\mathcal{F}_1\mathcal{L}_1 - \mathcal{F}_2\mathcal{L}_2)C}{\rho}\right)\frac{1}{\beta_1 - 1}\right]^{1-\beta_1} \\ &- \left[\frac{(\mathcal{F}_1 - \mathcal{F}_2)C}{(\rho - \alpha)\beta_1} D_0\right]^{\beta_1} \left[\left(I_{fixed} - \frac{2(\mathcal{F}_1\mathcal{L}_1 - \mathcal{F}_2\mathcal{L}_2)C}{\rho}\right)\frac{1}{\beta_1 - 1}\right]^{1-\beta_1} \end{aligned} \tag{3.34}$$

If we simplify,

$$\begin{aligned} I_0 < &\left[\frac{(\mathcal{F}_1 - \mathcal{F}_2)C}{(\rho - \alpha)\beta_1} D_0\right]^{\beta_1} \left(\frac{1}{\beta_1 - 1}\right)^{1-\beta_1} \left[\left(I_{flex} - \frac{2(\mathcal{F}_1\mathcal{L}_1 - \mathcal{F}_2\mathcal{L}_2)C}{\rho}\right)^{1-\beta_1} \right. \\ &\left. - \left(I_{fixed} - \frac{2(\mathcal{F}_1\mathcal{L}_1 - \mathcal{F}_2\mathcal{L}_2)C}{\rho}\right)^{1-\beta_1}\right] \end{aligned} \tag{3.35}$$

Right-hand side of inequality (3.35) can be defined as the upper bound for upfront cost. If I_0 is less than the upper bound, flexible design can be employed. Otherwise, the decision maker ought to adopt fixed design.

Another guideline can be derived in a similar way by solving inequality (3.35) for D_0. Instead of tracking option values, the decision maker can track D_0 and make decision accordingly.

In other words, if the decision maker is given I_0, I_{flex} and I_{fixed}, he/she prefers flexible design under the condition that

$$\left[\frac{I_0}{\left(I_{flex}-\frac{2(\mathcal{F}_1\mathcal{L}_1-\mathcal{F}_2\mathcal{L}_2)C}{\rho}\right)^{1-\beta_1}-\left(I_{fixed}-\frac{2(\mathcal{F}_1\mathcal{L}_1-\mathcal{F}_2\mathcal{L}_2)C}{\rho}\right)^{1-\beta_1}}\right]^{\frac{1}{\beta_1}} \left(\frac{1}{\beta_1-1}\right)^{\frac{\beta_1-1}{\beta_1}}\left(\frac{(\rho-\alpha)\beta_1}{(\mathcal{F}_1-\mathcal{F}_2)C}\right)<D_0 \tag{3.36}$$

In the following section, we demonstrate our mathematical model by solving a numerical example based on a real replenishment oiler.

Numerical Example

In Appendix 3.A, we give annual demand data ranging from 2004 to 2014. However, as stated in Assumption 1, it represents whole amount of fuel transported by all replenishment oilers in each year. We lack of a demand data set for a single replenishment oiler. Therefore, throughout this numerical example, we use hypothetical GBM parameters.

Let's assume that the receiving ships call for demand per 0.04 years (14.6 days). Suppose $\sigma = 0.03$, $\alpha = 0.05$ and $\rho = 0.06$ annually. Therefore, $\sigma = 0.0012$, $\alpha = 0.002$ and $\rho = 0.0024$ per 0.04 years. With these values, the conditions $\alpha - \sigma^2/2 = 0.0019 > 0$ and $\rho - \alpha = 0.0004 > 0$ are satisfied and β_1 is calculated as 1.199 by using Equation (3E.4) of Appendix 3.E.

Unlike the GBM parameters, we use as many real values as possible related to the replenishment oiler's design parameters in this numerical example. For this purpose, we take the replenishment oiler USS Passumpsic as an example, which was jumboized in 1960s (Wikipedia 2018b). It is stated in Wikipedia (2018b) that the its length was increased from 169 m to 196 m, its light ship mass was increased from 7,423 tons to 12,840 tons and its displacement was increased from 25,500 tons to 34,350 tons. (see also NavSource Online 2016). Thus, the parameters are written as $\Delta_1 = 25{,}500$ tons, $\Delta_2 = 34{,}350$ tons, $\mathcal{L}_1 = 7{,}423$ tons, $\mathcal{L}_2 = 12{,}840$ tons, $L_1 = 169$ m and $L_2 = 196$ m.

Wikipedia (2018b) expresses that its speed was 18.3 knots. In addition, we suppose that block coefficients of the replenishment oiler before and after jumboization are $B_1 = 0.93$ and $B_2 = 0.94$. Therefore, by using Equations (3.6) and (3.7),

$$m_1 = 4{,}977 \cdot 0.93^2 - 8{,}105 \cdot 0.93 + 4{,}456 = 1{,}223$$

$$n_1 = -10847 \cdot 0.93^2 + 12817 \cdot 0.93 - 6960 = -4{,}422$$

$$m_2 = 4{,}977 \cdot 0.94^2 - 8{,}105 \cdot 0.94 + 4{,}456 = 1{,}235$$

$$n_2 = -10847 \cdot 0.94^2 + 12817 \cdot 0.94 - 6960 = -4{,}496$$

With these values, we use Equation (3.5) to calculate

$$P_1 = \frac{25{,}500^{2/3} \cdot 18.3^3}{1{,}223 - 4{,}422 \cdot \dfrac{0.514 \cdot 18.3}{\sqrt{9.8 \cdot 169}}} = 26{,}421 \text{ kW}$$

$$P_2 = \frac{34{,}350^{2/3} \cdot 18.3^3}{1{,}235 - 4{,}496 \cdot \dfrac{0.514 \cdot 18.3}{\sqrt{9.8 \cdot 196}}} = 23{,}990 \text{ kW}$$

In Wikipedia (2018b), the installed power (or, the maximum power to propel the replenishment oiler) is given as 22,700 kW. Hence, it can be said that Equation (3.5) has good approximation. The maximum amounts of bunker fuel consumption in 0.04 year are given as

$$\frac{(0.0046 \cdot 26{,}421 + 0.2)2{,}800}{24 \cdot 18.3} = 776 \text{ tons}$$

$$\frac{(0.0046 \cdot 23{,}990 + 0.2)2{,}800}{24 \cdot 18.3} = 705 \text{ tons}$$

by assuming that the replenishment oiler traverses the distance $X = 2{,}800$ nautical miles in one direction in each 0.04 year. Note that these values are in tons and are needed to convert to gallons by using density value of bunker fuel. The type of bunker fuel is given as Navy Special Fuel Oil (NSFO, the U.S. Navy later switched to Naval Distillate Fuel, F-76. For details, see Tosh et al. 1992). Emergencies Science and Technology Division Environment Canada (2018) gives the density of NSFO as 0.9349 g/mL (or, 0.9349 kg/L). Since 1 oil barrel is equal to 159 liters (and 42 gallons), density of bunker fuel is found as 0.1486 tons/barrel. Thus, we obtain the maximum consumptions of bunker fuel per 0.04 years in gallon as

$$\frac{776}{0.1486} \cdot 42 = 219{,}285 \text{ gallons}$$

$$\frac{40{,}352}{0.1486} \cdot 42 = 199{,}137 \text{ gallons}$$

Finally, $\mathcal{F}_1$ and $\mathcal{F}_2$ are obtained by exploiting Equation (3.8) as

$$\mathcal{F}_1 = \frac{219{,}285}{25{,}500} = 8.6 \text{ gallons/ton}$$

$$\mathcal{F}_2 = \frac{199{,}137}{34{,}350} = 5.8 \text{ gallons/ton}$$

These numerical results indicate that jumboization is useful to bring about fuel saving for some demand values because while the replenishment oiler consumes 8.6 gallons of bunker fuel per ton displacement before jumboization, it consumes 5.8 gallons of it per ton displacement after jumboization.

Remaining parameters are cost of bunker fuel and jumboization cost. Nyserda (2017) states that $C = 2.46$ (\$/gallon) (note that with these values, the value of maximum amount of cargo fuel carried is approximately given as \$11.5M by stating that USS Passumpsic carries NSFO as well to replenish the receiving ships; see NavSource Online 2016) and Wildenberg (1996) gives jumboization cost as $I = \$20{,}000{,}000$ (we assume that it is $I_{flex} = \$20{,}000{,}000$). Therefore, by using Equation (3.21), we get $D^* = 14{,}537$ tons/year. Moreover, with these numerical values, option value at time 0 is obtained by using Equation (3.22) as $F_{flex}(D_0) = \$86{,}867{,}313$ with the assumption $D_0 = 7{,}000$ tons. On the other hand, if we assume $I_{fixed} = \$25{,}000{,}000$, it provides $F_{fixed}(D_0) = \$84{,}924{,}499$. Therefore, upfront cost for flexible design should not exceed $F_{flex}(D_0) - F_{fixed}(D_0) = \$1{,}942{,}813$. As for the other guideline regarding D_0, if I_0 is given as \$2,000,000, then initial demand value should not be less than 7,171 tons to prefer flexible design, derived by inequality (3.36). Given that $D_0 = 7{,}000$ tons, expected time duration until jumboization is calculated by using Equation (3.23) as $\tau = 14.62$ years.

Having determined D^* and stated relevant guidelines numerically regarding the choice between flexible and fixed designs, we want to verify that infinite life of option is not actually a deficiency for this problem. In the subsequent sections, we first demonstrate the binomial lattice calculations for 6 periods with each period equal to 0.04 years. After that, we present the result of the same problem, which is solved with a longer modeling horizon.

Option Valuation in Binomial Lattice with 6 Periods

We reiterate that $\sigma = 0.0012$ and $\alpha = 0.002$ per 0.04 years. Annual risk-free interest rate r is set as 0.02, which means $r = 0.0008$ per 0.04 years. Thus, the condition $\rho - r = 0.0016 > 0$

is satisfied. In the previous section, we assume that $D_0 = 7{,}000$ tons while calculating option values. In the binomial lattice calculations, for the purpose of demonstration, we adopt that $D_0 = 600{,}000$ tons because we would like to show some nodes of the lattices in which jumboization investment appears. $D_0 = 7{,}000$ tons is too low for 6 periods modeling horizon with 0.04 years granularity to see a lattice node in which investment is made.

Since the unit of σ is %/0.04 years, $\Delta t = 1$ is taken into account to calculate u and d factors. By using Equations (3.24) and (3.25), we determine $u = 1.0012$ and $d = 0.9988$. The condition $d < 1 + r < u$ holds and risk-neutral probability for up movement is calculated by using Equation (3.27) as $q = 0.833$.

Since $\Delta t = 1$ (0.04 years) and $T = 6$, labels of time points start with 0 and goes through 6. Table 3.2 shows all lattices created in three steps. For all lattices, horizontal move towards right (from t to $t + 1$ with the same k) represents up movement for a node. On the other hand, the movement from t to $t + 1$ and from k to $k + 1$ represents down movement. Table 3.2(a) is demand evolution lattice. Table 3.2(b) presents the evolution of the replenishment oiler's value in the case that it is already jumboized at time 0. Table 3.2(c) presents the evolution of replenishment oiler's value, but with jumboization option.

Throughout the numerical example, we will demonstrate some of the calculations in the binomial lattices. For easiness, we first present the calculations $2(\mathcal{F}_1\mathcal{L}_1 - \mathcal{F}_2\mathcal{L}_2)C = -52{,}172$ and $(\mathcal{F}_1 - \mathcal{F}_2)C = 6.89$ because they are repeatedly used.

For Table 3.2(b), the values at the terminating nodes are all 0. For $t = 5$ and $k = 1$, the value is calculated by using Equation (3.28) as

$$\mathcal{V}_{(5,1)} = \frac{[-52{,}172\ + 6.89 \cdot 603{,}611]}{1 + 0.0008} = \$4{,}105{,}327$$

For $t = 4$ and $k = 1$, the value is calculated by using Equation (3.29) as

$$\mathcal{V}_{(4,1)} = \frac{[-52{,}172 + 6.89 \cdot 602{,}887]}{1 + 0.0008} + \frac{4{,}105{,}327 \cdot 0.833 + 4{,}095{,}361 \cdot (1 - 0.833)}{1 + 0.0008} = \$8{,}200{,}725$$

For Table 3.2(c), the nodes in bold are those in which the decision maker chooses to invest. For $t = 5$ and $k = 1$, the value is calculated by using Equation (3.30) as

$$\hat{\mathcal{V}}_{(5,1)} = \max\{4{,}105{,}327 - 20{,}000{,}000; 0\} = 0$$

For $t = 3$ and $k = 1$, the value is calculated by using Equation (3.31) as

$$\hat{\mathcal{V}}_{(3,1)} = \max\left\{12{,}286{,}209 - 20{,}000{,}000; \frac{0 \cdot 0.833 + 0 \cdot (1 - 0.833)}{1 + 0.0008}\right\} = 0$$

Similarly, for $t = 1$ and $k = 1$, the value is calculated as

$$\hat{\mathcal{V}}_{(1,1)} = \max\left\{20{,}427{,}514 - 20{,}000{,}000; \frac{0 \cdot 0.833 + 0 \cdot (1 - 0.833)}{1 + 0.0008}\right\} = \$427{,}514$$

Table 3.2 *Result of evaluation of jumboization option with the binomial lattices*

a) Growth of demand with respect to time (tons)

k	$t = 0$	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$
1	600,000	600,720	601,442	602,164	602,887	603,611	604,336
2		599,280	600,000	600,720	601,442	602,164	602,887
3			598,562	599,280	600,000	600,720	601,442
4				597,844	598,562	599,280	600,000
5					597,127	597,844	598,562
6						596,411	597,127
7							595,696

b) Growth of replenishment oiler's value with jumboization in place with respect to time ($)

k	$t = 0$	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$
1	24,483,369	20,427,514	16,361,800	12,286,209	8,200,725	4,105,327	0
2		20,377,923	16,322,079	12,256,383	8,180,817	4,095,361	0
3			16,282,454	12,226,629	8,160,956	4,085,419	0
4				12,196,945	8,141,144	4,075,501	0
5					8,121,379	4,065,607	0
6						4,055,736	0
7							0

c) Growth of replenishment oiler's value with jumboization option with respect to time ($)

k	$t = 0$	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$
1	**4,483,369**	**427,514**	0	0	0	0	0
2		**377,923**	0	0	0	0	0
3			0	0	0	0	0
4				0	0	0	0
5					0	0	0
6						0	0
7							0

meaning that jumboization takes place in this node. For $t = 0$ and $k = 1$, the value is calculated as

$$\hat{\mathcal{V}}_{(0,1)} = \max\left\{24{,}483{,}369 - 20{,}000{,}000; \frac{427{,}514 \cdot 0.833 + 377{,}923 \cdot (1 - 0.833)}{1 + 0.0008}\right\}$$

$$= \$4{,}483{,}369$$

meaning that the investment takes place in this node as well.

Determining $D^*(t)$ in Binomial Lattice Calculations

Having completed option evaluation in the binomial lattices, we proceed to determine threshold demand levels in each t. We list below the results of calculations for each t and make the relevant explanations. For each $t \in [0, 1, 2, 3, 4, 5]$, we terminate the iterations to seek for $D^*(t)$ when the difference between left-hand side and right-hand side of Equation (3.32) is below 1.

For $t = 0$, the lattices given in Table 3.3 turn out to be the final lattices in which the continuation value and the immediate benefit from jumboization at $t = 0$ are sufficiently close to each other. For $t = 0$ and $k = 1$, the following values are calculated as the immediate benefit and the continuation value:

$$\mathcal{V}_{(0,1)} - I = 20{,}000{,}000.0201 - 20{,}000{,}000 = 0.0201$$

$$\frac{\hat{\mathcal{V}}_{(1,1)}q + \hat{\mathcal{V}}_{(1,2)}(1 - q)}{1 + r\Delta t} = \frac{0 \cdot 0.833 + 0 \cdot (1 - 0.833)}{1 + 0.0008} = 0$$

It indicates that $D^*(0) = 491{,}511.83$ tons/0.04 years.

Table 3.3 *Result of the last iteration in which $D^*(0)$ is found*

a) Growth of demand with respect to time (tons)

k	$t=0$	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$
1	491,511.83	492,102	492,693	493,284	493,877	494,470	495,063
2		490,922	491,512	492,102	492,693	493,284	493,877
3			490,334	490,922	491,512	492,102	492,693
4				489,746	490,334	490,922	491,512
5					489,158	489,746	490,334
6						488,572	489,158
7							487,986

b) Growth of replenishment oiler's value with jumboization in place with respect to time ($)

k	$t=0$	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$
1	20,000,000	16,686,887	13,365,705	10,036,441	6,699,078	3,353,602	0
2		16,646,263	13,333,167	10,012,007	6,682,769	3,345,439	0
3			13,300,706	9,987,633	6,666,500	3,337,294	0
4				9,963,317	6,650,270	3,329,169	0
5					6,634,079	3,321,064	0
6						3,312,978	0
7							0

c) Growth of replenishment oiler's value with jumboization option with respect to time ($)

k	$t=0$	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$
1	0.0201	0	0	0	0	0	0
2		0	0	0	0	0	0
3			0	0	0	0	0
4				0	0	0	0
5					0	0	0
6						0	0
7							0

The lattices given in Table 3.4 are the final lattices for $t=1$. In Table 3.4(c), for $t=1$ and $k=1$, while the immediate benefit is calculated as 0.1239, the continuation value is 0. It results in $D^*(1) = 588{,}306.51$ tons/0.04 years.

Table 3.4 *Result of the last iteration in which $D^*(1)$ is found*

a) Growth of demand with respect to time (tons)

k	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$
1	588,306.51	589,013	589,720	590,428	591,137	591,847
2		587,601	588,307	589,013	589,720	590,428
3			586,896	587,601	588,307	589,013
4				586,192	586,896	587,601
5					585,489	586,192
6						584,787

b) Growth of replenishment oiler's value with jumboization in place with respect to time ($)

k	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$
1	20,000,000	16,019,378	12,029,085	8,029,102	4,019,413	0
2		15,980,479	11,999,875	8,009,606	4,009,653	0
3			11,970,735	7,990,156	3,999,917	0
4				7,970,753	3,990,203	0
5					3,980,514	0
6						0

c) Growth of replenishment oiler's value with jumboization option with respect to time ($)

k	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$
1	0.1239	0	0	0	0	0
2		0	0	0	0	0
3			0	0	0	0
4				0	0	0
5					0	0
6						0

For $t = 2$, the iterations are terminated when the lattices showed in Table 3.5 are obtained. In Table 3.5(c), for $t = 2$ and $k = 1$, while the immediate benefit is calculated as 0.1639, the continuation value is 0. It results in $D^*(2) = 733{,}497.02$ tons/0.04 years.

For $t = 3$, the lattices given in Table 3.6 turn out to be the final lattices when the iterations are terminated. In Table 3.6(c), for $t = 3$ and $k = 1$, the immediate benefit and the continuation value are calculated as 0.1735 and 0, respectively. It results in $D^*(3) = 975{,}479.19$ tons/0.04 years.

Table 3.5 *Result of the last iteration in which $D^*(2)$ is found*

a) Growth of demand with respect to time (tons)

k	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$
1	733,497.02	734,378	735,260	736,142	737,026
2		732,617	733,497	734,378	735,260
3			731,739	732,617	733,497
4				730,861	731,739
5					729,985

b) Growth of replenishment oiler's value with jumboization in place with respect to time ($)

k	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$
1	20,000,000	15,018,136	10,024,196	5,018,158	0
2		14,981,761	9,999,917	5,006,004	0
3			9,975,696	4,993,879	0
4				4,981,783	0
5					0

c) Growth of replenishment oiler's value with jumboization option with respect to time ($)

k	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$
1	0.1639	0	0	0	0
2		0	0	0	0
3			0	0	0
4				0	0
5					0

For $t = 4$, we terminate the iterations when the lattices presented in Table 3.7 are obtained. In Table 3.7(c), for $t = 4$ and $k = 1$, the immediate benefit and the continuation value are calculated as 0.0482 and 0, respectively. It gives $D^*(4) = 1{,}459{,}440.5$ tons/0.04 years.

Table 3.8 shows the final lattices when the iterations to seek for $D^*(5)$ are terminated. In Table 3.8(c), for $t = 5$ and $k = 1$, the immediate benefit and the continuation value are calculated as 0.0278 and 0, respectively. It shows $D^*(5) = 2{,}911{,}318$ tons/year.

Table 3.6 *Result of the last iteration in which $D^*(3)$ is found*

a) Growth of demand with respect to time (tons)

k	$t = 3$	$t = 4$	$t = 5$	$t = 6$
1	975,479.19	976,650	977,823	978,997
2		974,309	975,479	976,650
3			973,141	974,309
4				971,974

b) Growth of replenishment oiler's value with jumboization in place with respect to time ($)

k	$t = 3$	$t = 4$	$t = 5$	$t = 6$
1	20,000,000	13,349,426	6,682,769	0
2		13,317,176	6,666,625	0
3			6,650,519	0
4				0

c) Growth of replenishment oiler's value with jumboization option with respect to time ($)

k	$t = 3$	$t = 4$	$t = 5$	$t = 6$
1	0.1735	0	0	0
2		0	0	0
3			0	0
4				0

Table 3.7 *Result of the last iteration in which $D^*(4)$ is found*

a) Growth of demand with respect to time (tons)

k	$t = 4$	$t = 5$	$t = 6$
1	1,459,440.5	1,461,193	1,462,947
2		1,457,690	1,459,441
3			1,455,942

b) Growth of replenishment oiler's value with jumboization in place with respect to time ($)

k	$t = 4$	$t = 5$	$t = 6$
1	20,000,000.0482	10,012,049	0
2		9,987,924	0
3			0

c) Growth of replenishment oiler's value with jumboization option with respect to time ($)

k	$t = 4$	$t = 5$	$t = 6$
1	0.0482	0	0
2		0	0
3			0

Table 3.8 *Result of the last iteration in which $D^*(5)$ is found*

a) Growth of demand with respect to time (tons)

k	$t = 5$	$t = 6$
1	2,911,318	2,914,814
2		2,907,827

b) Growth of replenishment oiler's value with jumboization in place with respect to time ($)

k	$t = 5$	$t = 6$
1	20,000,000.0278	0
2		0

c) Growth of replenishment oiler's value with jumboization option with respect to time ($)

k	$t = 5$	$t = 6$
1	0.0278	0
2		0

When all parameter values are kept the same except $T = 14$ years ($T_{\Delta t} = 350$ periods), the results depicted in Figure 3.1 are obtained. For each t, we terminate the iterations to seek for $D^*(t)$ when the difference between left-hand side and right-hand side of Equation (3.32) is below 0.01. As can be seen in Figure 3.1, $D^*(0) = 14{,}900$ tons/0.04 years. Since D^* and $D^*(0)$ are close to each other, we justify that infiniteness assumption of option life and service life of the replenishment oiler is not deficient because as the results show, even 14 years is close to infinity.

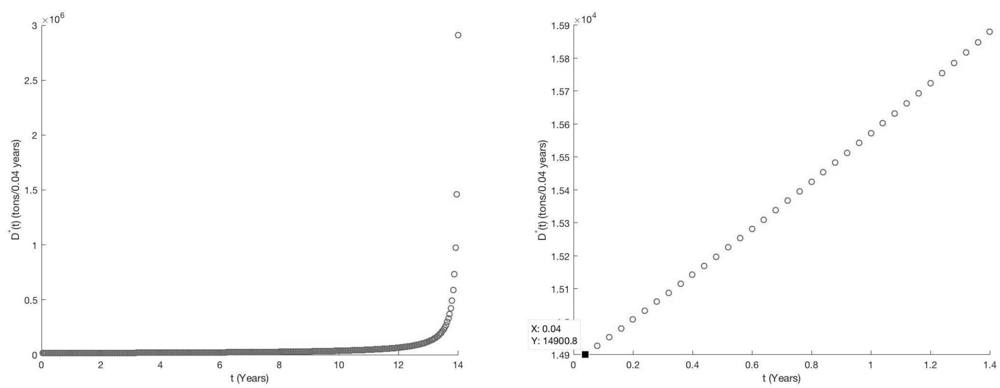

Figure 3.1 *$D^*(t)$ values when T is 14 years (Right picture zooms initial part)*

Discussions on Assumptions 3 and 4 for Possible Generalizations

In Assumption 3, we state that the replenishment oiler makes round trip between its port and the location of the receiving ships. Therefore, we assume that the replenishment oiler always moves between two specified ports. As can be derived from our mathematical framework, the existence of two specified ports is not necessary. The replenishment oiler can depart from a port to replenish the receiving ships in one location, and then it can return to a different port. When a different set of receiving ships, which is at a different location, calls for replenishment, the replenishment oiler moves towards these ships to fulfill the task. However, the distance between ports and the locations should still be X. Since equality of distance between all ports and locations do not reflect the reality, we do not consider it in this study.

Assumption 4 states that the replenishment oiler does not change its speed throughout the modeling horizon. It can be generalized to the case of four different speeds. The replenishment oiler can reduce its speed after jumboization for the purpose of fuel saving (Lewis et al. 1977). Moreover, it can reduce the speed while transporting the fuel to the receiving ships for the purpose of fuel saving again. Thus, this combination results in four different speeds. Although our framework allows to adopt this generalization, we do not prefer it because there exists an ambiguity regarding who determines the speed of the replenishment oiler.

Concluding Remarks and Future Researches

In this chapter, we show how to quantify the value of jumboization option for U.S. Navy transportation ships by particularly focusing on replenishment oilers. Having modeled that jumboization brings about fuel cost saving, we derive expected time of jumboization investment and its value contingent upon the uncertain demand factor. It is shown that analytical framework with infinite life of replenishment oiler assumption and its discrete counterpart model give very close results, which signifies that this assumption is not an inadequacy for the model. A managerial guideline is discovered regarding the choice between flexible and fixed designs and it points out that relatively low demand values at the initial stage of design should be accepted as signal to adopt fixed design. Future studies of this chapter could involve the abandonment and purchasing options of the replenishment oiler. Moreover, another uncertain factor and its corresponding stochastic process could be taken into account to build the underlying framework.

Appendix 3.A Statistical Validation of GBM Assumption

Table 3.A.1 collects relevant data, which is given in Shannon (2014) and similar reports published before 2014.

Table 3.A.1 *Amount of fuel transported by replenishment oilers*

Year	Transported Fuel Amount	Year	Transported Fuel Amount	Year	Transported Fuel Amount
2004	428,000,000	2008	549,181,418	2012	555,753,996
2005	466,000,000	2009	710,041,752	2013	523,530,000
2006	579,312,543	2010	1,154,792,960	2014	459,529,812
2007	581,899,405	2011	583,602,984		

Line graph of the given data set is depicted in Figure 3.A.1:

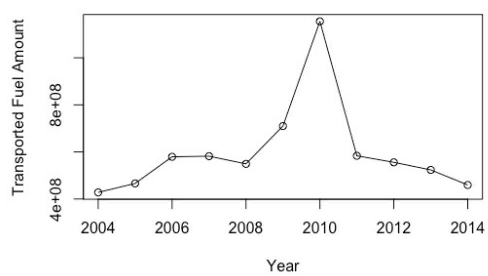

Figure 3.A.1 *Line graph of amount of fuel transported by replenishment oilers*

To test if a given data set fits GBM process, we need to convert the data set to the corresponding successive log ratios. Let $\theta_y = \ln \phi_y - \ln \phi_{y-1}$ where ϕ_y is the fuel amount transported for year y, $2005 \leq y \leq 2014$. Table 3.A.2 presents the result of the conversion.

Table 3.A.2 *Log ratios*

y	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014
θ_y	0.085	0.218	0.004	-0.058	0.257	0.486	-0.682	-0.049	-0.060	-0.130

Having obtained log ratios, two properties should be checked (see, e.g., Marathe and Ryan 2005): (i) set of θ_y is normally distributed and (ii) θ_{y+1} is independent of θ_y. In statistical theory, there are several methods to test the normality of a given data set. Drawing the histogram and QQ plot are among the easiest methods. Figure 3.A.2 shows the histogram and QQ plot of θ_y. QQ plot compares the quantiles of the sample and theoretical quantiles of normal distribution. If those points lie over $y = x$ line, it means that two distributions are the same. If those points lie over a linear line, it is interpreted that distributions are linearly related (Linearly related means two distributions are the same, but they have different parameters). Since interpreting histogram and QQ plot may not be accurate to reach a conclusion, we need to proceed to statistical tests.

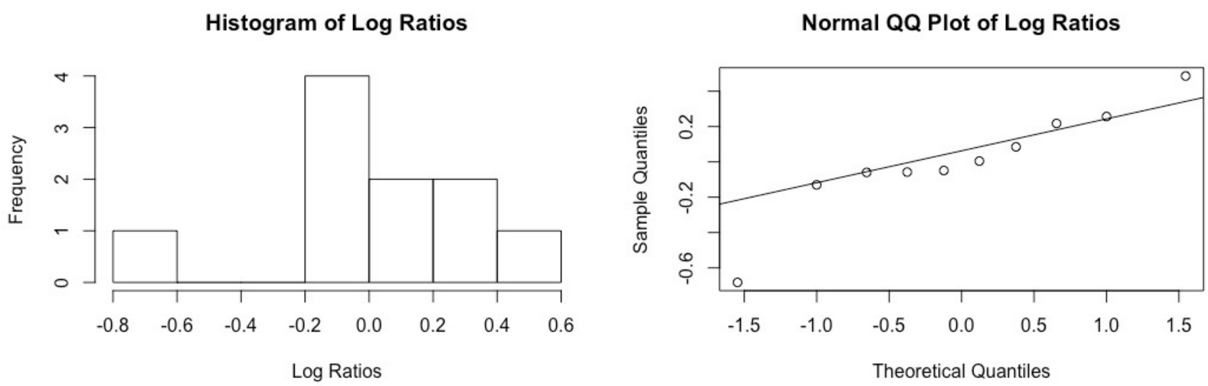

Figure 3.A.2 *Histogram and QQ plot of log ratios*

In literature, there exist several statistical tests to check the normality. Shapiro-Wilk test for normality and Andersen-Darling test for normality are among the most frequently utilized tests. Both tests use the following hypotheses:

- H_0: The observed distribution fits the normal distribution
- H_a: The observed distribution does not fit the normal distribution

Shapiro-Wilk test for normality results in test statistic 0.9061 and p-value 0.2576. Since p-value is greater than significance level (set as 0.05), we do not have enough evidence to reject the null hypotheses. Similarly, Andersen-Darling test for normality gives test statistic 0.4897 and p-value 0.1685. Since p-value is larger than significance level, we cannot reject the null hypotheses. Both test results allow us to claim that set of θ_y fits the normal distribution.

Testing if a given data set represents independent increment can be succeeded with Chi-square test for independence (see, e.g., Marathe and Ryan 2005 and Ross 2011) with the following hypotheses:

- H_0: The observed distribution has the independent increments
- H_a: The observed distribution does not have the independent increments

The basic idea of this test is to create possible states for each year and observe if there is a dependency between the successive states. It is known that GBM process implies independent increments, which means the state of year $y + 1$ should be independent of the state of year y. In other words, if the process is in state i in year y, there should be equal probability of being in any state j in year $y + 1$. To test this property, we create two-way table, whose rows and columns include the defined states. For this purpose, we create the following five states:

- $\theta_y \leq -0.1 \Rightarrow$ state 1 (years 2011, 2014)
- $-0.1 < \theta_y \leq -0.05 \Rightarrow$ state 2 (years 2008, 2013)
- $-0.05 < \theta_y \leq 0.05 \Rightarrow$ state 3 (years 2007, 2012)
- $0.05 < \theta_y \leq 0.25 \Rightarrow$ state 4 (years 2005, 2006)
- $0.25 < \theta_y \Rightarrow$ state 5 (years 2009, 2010)

Critical point in defining these states is that each state should have approximately equal number of years. The years given in parentheses show those which fall in the relevant states. Having determined the states, two-way table, Table 3.A.3, is created by enumerating how many times state j is followed by state i.

Table 3.A.3 *States*

States	1	2	3	4	5
1	0	0	1	0	0
2	1	0	0	0	1
3	0	2	0	0	0
4	0	0	1	1	0
5	1	0	0	0	1

Each cell (denoted by O_{ij} where i is index for rows and j is index for columns) in above table represents the number of times state i is followed by state j. The next step is to determine the expected values for each cell. Expected value for a cell is determined as

$$E_{ij} = \frac{\sum Row_i \cdot \sum Column_j}{N} \tag{3A.1}$$

where $\sum Row_i$ is the sum of values in row i of two-way table, $\sum Column_j$ is the sum of the values in column j of two-way table and N is the sum of all values in two-way table. Table 3.A.4 presents the expected values for each cell.

Table 3.A.4 *Expected values for each cell*

States	1	2	3	4	5
1	0.222	0.222	0.222	0.111	0.222
2	0.444	0.444	0.444	0.222	0.444
3	0.444	0.444	0.444	0.222	0.444
4	0.444	0.444	0.444	0.222	0.444
5	0.444	0.444	0.444	0.222	0.444

In the final step, Chi-square test statistic is calculated as

$$\chi^2 = \sum_{ij} \frac{\left(E_{ij} - O_{ij}\right)^2}{E_{ij}} \tag{3A.2}$$

and it is given as 20.25. We need to compare it with the critical value. Degrees of freedom in Chi-square test is found by the (number of rows - 1) multiplied by (number of columns - 1) in two-way table, and it gives 16 in our case. With significance level of 0.05 and degrees of freedom 16, critical test value is found as 7.96. Moreover, *p*-value is found as 0.209. Since *p*-value is greater than 0.05, we do not have enough evidence to reject the null hypothesis.

As a result, it can be said that fuel amount data follows GBM process.

Appendix 3.B Admiralty Method

Resistance against a ship at sea may be written as

$$R \propto \varrho W S^{\gamma} \tag{3B.1}$$

where ϱ is the density of seawater (kg/m^3), W is the wetted surface area of the ship (m^2) and S is the speed of the ship (knot), as defined in Table 3.1 (We note that when $\gamma = 2$, the resistance unit turns out to be Newton). W is proportional to $\Delta^{2/3}$ because displacement of a ship is in fact the mass of seawater displaced and $\Delta^{2/3}$gives the wetted surface area by assuming that density of seawater is only 1. Therefore, for a constant density,

$$R \propto \Delta^{2/3} S^{\gamma} \tag{3B.2}$$

We know that power can be expressed as $P \propto RS$ or $P \propto \Delta^{2/3} S^{\gamma+1}$. Thus,

$$P = \frac{\Delta^{2/3} S^{\gamma+1}}{\text{a coefficient}} \tag{3B.3}$$

For most merchant ship, γ is taken to be 2. Hence, Equation (3B.3) becomes

$$P = \frac{\Delta^{2/3} S^{3}}{\mathcal{A}} \tag{3B.4}$$

where $\mathcal{A}$ is Admiralty coefficient, as defined in Equation (3.2) (Stokoe 2003 and HubPages 2010).

Appendix 3.C Fubini's Theorem

Fubini's theorem states that

$$E\left[\int_0^\infty D_t e^{-\rho t} dt\right] = \int_0^\infty E[D_t e^{-\rho t}]dt\,, \quad \text{if} \int_0^\infty E|D_t e^{-\rho t}|dt < \infty \tag{3C.1}$$

It is clear that $D_t e^{-\rho t} \geq 0$ because of the GBM properties. Thus,

$$\int_0^\infty E|D_t e^{-\rho t}|dt = \int_0^\infty E[D_t]e^{-\rho t}dt \tag{3C.2}$$

By Appendix 3.D, we know the solution of D_t. Therefore, to find $E[D_t]$, write as

$$E[D_t] = E\left[D_0 e^{\left(\alpha - \frac{\sigma^2}{2}\right)t + \sigma z_t}\right] = D_0 e^{\left(\alpha - \frac{\sigma^2}{2}\right)t} E[e^{\sigma z_t}] \tag{3C.3}$$

We know that $z_t \sim N(0, t)$ and moment generating function of any normally distributed random variable $X \sim N(\mu, \sigma^2)$ is given by

$$E\left[e^{\psi X}\right] = e^{\mu\psi + \frac{\psi^2 \sigma^2}{2}} \tag{3C.4}$$

Therefore,

$$E[D_t] = D_0 e^{\alpha t} \tag{3C.5}$$

Since jumboization is done when demand is at the level of D_x, D_0 can be replaced with D_x. Thus, $E[D_t] = D_x e^{\alpha t}$. As a result, we can write

$$\int_0^\infty E|D_t e^{-\rho t}|dt = D_x \left(\frac{e^{(\alpha-\rho)t}}{\alpha - \rho}\right)\Bigg|_0^\infty = \frac{D_x}{\rho - \alpha} < \infty \tag{3C.6}$$

Hence, change of orders of integral and expectation operators is allowed.

Appendix 3.D Solution of D_t

Let's rewrite the differential form of D_t, stated in Equation (3.1), as follows:

$$\frac{dD_t}{D_t} = \alpha dt + \sigma dz \tag{3D.1}$$

Integrating both sides leads to

$$\int_0^t \frac{dD_t}{D_t} = \alpha t + \sigma z_t \tag{3D.2}$$

$\frac{dD_t}{D_t}$ seems an ordinary differential of D_t, but D_t is itself in Ito representation. Therefore, we need to apply Ito's formula as ordinary differentiation does not work. Let's take $\ln D_t$ function. Thus,

$$d(\ln D_t) = \frac{1}{D_t} dD_t + \frac{1}{2}\left(-\frac{1}{D_t^2}\right)(dD_t)^2 \tag{3D.3}$$

We need to find the expression for $(dD_t)^2$. Write it as

$$(dD_t)^2 = \alpha^2 D_t^2 (dt)^2 + \sigma^2 D_t^2 (dz)^2 + 2\alpha\sigma D_t^2 dzdt \tag{3D.4}$$

The first term has $(dt)^2$, the second term has dt and the third term has $(dt)^{\frac{3}{2}}$ because $dz = \epsilon\sqrt{dt}$. In this case, $(dt)^2$ and $(dt)^{\frac{3}{2}}$ can be eliminated because they go to 0 faster than dt when $dt \rightarrow 0$. Thus, $(dD_t)^2 = \sigma^2 D_t^2 dt$. When this expression is plugged into Equation (3D.3), it is obtained

$$\frac{dD_t}{D_t} = d(\ln D_t) + \frac{\sigma^2}{2} dt \tag{3D.5}$$

Therefore, Equation (3D.2) can be written as

$$\int_0^t \left(d(\ln D_t) + \frac{\sigma^2}{2} dt\right) = \int_0^t d(\ln D_t) + \frac{\sigma^2 t}{2} = \alpha t + \sigma z_t \tag{3D.6}$$

It can be simplified as

$$\ln \frac{D_t}{D_0} = \left(\alpha - \frac{\sigma^2}{2}\right) t + \sigma z_t \tag{3D.7}$$

and we get

$$D_t = D_0 e^{\left(\alpha - \frac{\sigma^2}{2}\right)t + \sigma z_t} \tag{3D.8}$$

Appendix 3.E Solving $F(D)$

Let's plug the solution $F(D) = AD^{\beta}$ into Equation (3.18). By using the derivatives $F'(D) = A\beta D^{\beta-1}$ and $F''(D) = A\beta(\beta-1)D^{\beta-2}$, it can be written as

$$\frac{1}{2}\sigma^2 A\beta(\beta-1)D^{\beta} + \alpha A\beta D^{\beta} - \rho AD^{\beta} = 0 \tag{3E.1}$$

With simplification, it is obtained

$$AD^{\beta}\left[\frac{1}{2}\sigma^2\beta(\beta-1) + \alpha\beta - \rho\right] = 0 \tag{3E.2}$$

Since $A \neq 0$ and $D^{\beta} \neq 0$, it is written

$$\frac{1}{2}\sigma^2\beta^2 + \left(\alpha - \frac{1}{2}\sigma^2\right)\beta - \rho = 0 \tag{3E.3}$$

This is a second-order polynomial function. Therefore, one can get two distinct solutions as

$$\beta_{1,2} = \frac{1}{2} - \frac{\alpha}{\sigma^2} \pm \sqrt{\left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)^2 + \frac{2\rho}{\sigma^2}} \tag{3E.4}$$

It is verified in Dixit and Pindyck (1994) that $\beta_1 > 1$ and $\beta_2 < 0$.

Appendix 3.F Finding D^* and $F(D)$

Equations (3.19) and (3.20) are written as

$$A_1 D^{*\beta_1} = \frac{2(\mathcal{F}_1\mathcal{L}_1 - \mathcal{F}_2\mathcal{L}_2)C}{\rho} + \frac{(\mathcal{F}_1 - \mathcal{F}_2)C}{\rho - \alpha} D^* - I \tag{3F.1}$$

$$A_1 \beta_1 D^{*\beta_1 - 1} = \frac{(\mathcal{F}_1 - \mathcal{F}_2)C}{\rho - \alpha} \tag{3F.2}$$

From Equation (3F.2),

$$A_1 = \frac{(\mathcal{F}_1 - \mathcal{F}_2)C}{(\rho - \alpha)\beta_1} D^{*1-\beta_1} \tag{3F.3}$$

When Equation (3F.3) is plugged into Equation (3F.1) and it is simplified, one obtains

$$D^* = \left(I - \frac{2(\mathcal{F}_1\mathcal{L}_1 - \mathcal{F}_2\mathcal{L}_2)C}{\rho}\right) \frac{\beta_1(\rho - \alpha)}{(\beta_1 - 1)(\mathcal{F}_1 - \mathcal{F}_2)C} \tag{3F.4}$$

By Equation (3F.4), $F(D)$ is derived as

$$F(D) = \left[\frac{(\mathcal{F}_1 - \mathcal{F}_2)C}{(\rho - \alpha)\beta_1} D\right]^{\beta_1} \left[\left(I - \frac{2(\mathcal{F}_1\mathcal{L}_1 - \mathcal{F}_2\mathcal{L}_2)C}{\rho}\right) \frac{1}{\beta_1 - 1}\right]^{1-\beta_1} \tag{3F.5}$$

Appendix 3.G Finding $\frac{\partial \beta_1}{\partial \sigma}$

Let's write β_1 as

$$\beta_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\frac{1}{4} - \frac{\alpha}{\sigma^2} + \frac{\alpha^2}{\sigma^4} + \frac{2\rho}{\sigma^2}} = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \left(\frac{1}{4} - \frac{\alpha}{\sigma^2} + \frac{\alpha^2}{\sigma^4} + \frac{2\rho}{\sigma^2}\right)^{1/2} \quad \text{(3G.1)}$$

Then,

$$\frac{\partial \beta_1}{\partial \sigma} = \frac{2\alpha}{\sigma^3} + \frac{\frac{2\alpha}{\sigma^3} - \frac{4\alpha^2}{\sigma^5} - \frac{4\rho}{\sigma^3}}{2\sqrt{\frac{1}{4} - \frac{\alpha}{\sigma^2} + \frac{\alpha^2}{\sigma^4} + \frac{2\rho}{\sigma^2}}} \quad \text{(3G.2)}$$

If $\frac{2\alpha}{\sigma^3} - \frac{4\alpha^2}{\sigma^5} - \frac{4\rho}{\sigma^3} > 0$ (or $\alpha(\sigma^2 - 2\alpha) - 2\rho\sigma^2 > 0$), then right-hand side of Equation (3G.2) turns out to be positive. However, due to the technical assumption $\alpha - \sigma^2/2 > 0$, we can conclude that $\alpha(\sigma^2 - 2\alpha) - 2\rho\sigma^2 < 0$. Therefore, we need to expand the right-hand side of Equation (3G.2). It can be written as

$$\frac{\partial \beta_1}{\partial \sigma} = \frac{4\alpha\sqrt{\frac{1}{4} - \frac{\alpha}{\sigma^2} + \frac{\alpha^2}{\sigma^4} + \frac{2\rho}{\sigma^2}} + \left(2\alpha - \frac{4\alpha^2}{\sigma^2} - 4\rho\right)}{2\sigma^3\sqrt{\frac{1}{4} - \frac{\alpha}{\sigma^2} + \frac{\alpha^2}{\sigma^4} + \frac{2\rho}{\sigma^2}}} \quad \text{(3G.3)}$$

Since the denominator is positive, it suffices to check the sign of numerator. Let

$$x = 4\alpha\sqrt{\frac{1}{4} - \frac{\alpha}{\sigma^2} + \frac{\alpha^2}{\sigma^4} + \frac{2\rho}{\sigma^2}} \quad \text{(3G.4)}$$

$$y = \left(2\alpha - \frac{4\alpha^2}{\sigma^2} - 4\rho\right) \quad \text{(3G.5)}$$

We know that $x > 0$ and $y < 0$. If we show that $x^2 - y^2 = (x - y)(x + y) > 0$, then we prove $x + y > 0$. Thus,

$$\left(4\alpha\sqrt{\frac{1}{4}-\frac{\alpha}{\sigma^2}+\frac{\alpha^2}{\sigma^4}+\frac{2\rho}{\sigma^2}}\right)^2-\left(2\alpha-\frac{4\alpha^2}{\sigma^2}-4\rho\right)^2$$
$$=16\alpha^2\left(\frac{1}{4}-\frac{\alpha}{\sigma^2}+\frac{\alpha^2}{\sigma^4}+\frac{2\rho}{\sigma^2}\right)-4\alpha^2+\frac{16\alpha^3}{\sigma^2}-\frac{32\alpha^2\rho}{\sigma^2}+16\alpha\rho$$
$$-\frac{16\alpha^4}{\sigma^4}-16\rho^2 \qquad (3G.6)$$
$$=4\alpha^2-\frac{16\alpha^3}{\sigma^2}+\frac{16\alpha^4}{\sigma^4}+\frac{32\alpha^2\rho}{\sigma^2}-4\alpha^2+\frac{16\alpha^3}{\sigma^2}-\frac{32\alpha^2\rho}{\sigma^2}$$
$$+16\alpha\rho-\frac{16\alpha^4}{\sigma^4}-16\rho^2=16\rho(\alpha-\rho)<0$$

Since $(x-y)(x+y)<0$, we say that $x+y<0$. Therefore, $\frac{\partial\beta_1}{\partial\sigma}<0$.

Appendix 3.H Finding $\frac{\partial D^*}{\partial \beta_1}$

Let's rewrite D^* in the form of

$$D^* = \frac{\left(I - \frac{2(\mathcal{F}_1\mathcal{L}_1 - \mathcal{F}_2\mathcal{L}_2)C}{\rho}\right)\frac{(\rho - \alpha)}{(\mathcal{F}_1 - \mathcal{F}_2)C}\beta_1}{(\beta_1 - 1)} \tag{3H.1}$$

Therefore,

$$\frac{\partial D^*}{\partial \beta_1} = \frac{-\left(I - \frac{2(\mathcal{F}_1\mathcal{L}_1 - \mathcal{F}_2\mathcal{L}_2)C}{\rho}\right)\frac{(\rho - \alpha)}{(\mathcal{F}_1 - \mathcal{F}_2)C}}{(\beta_1 - 1)^2} \tag{3H.2}$$

Since we assume that $I - \frac{2(\mathcal{F}_1\mathcal{L}_1 - \mathcal{F}_2\mathcal{L}_2)C}{\rho} > 0$, we conclude $\frac{\partial D^*}{\partial \beta_1} < 0$.

Appendix 3.I Finding $\frac{\partial D^*}{\partial X}$

Let's rewrite D^* in the form of

$$D^* = \frac{I\beta_1(\rho-\alpha)}{(\beta_1-1)C}\frac{1}{(\mathcal{F}_1-\mathcal{F}_2)} - \frac{2\beta_1(\rho-\alpha)}{\rho(\beta_1-1)}\frac{\mathcal{F}_1\mathcal{L}_1-\mathcal{F}_2\mathcal{L}_2}{\mathcal{F}_1-\mathcal{F}_2} \tag{3I.1}$$

Then,

$$\frac{\partial D^*}{\partial X} = \frac{I\beta_1(\rho-\alpha)}{(\beta_1-1)C}\frac{\partial\frac{1}{(\mathcal{F}_1-\mathcal{F}_2)}}{\partial X} - \frac{2\beta_1(\rho-\alpha)}{\rho(\beta_1-1)}\frac{\partial\frac{\mathcal{F}_1\mathcal{L}_1-\mathcal{F}_2\mathcal{L}_2}{\mathcal{F}_1-\mathcal{F}_2}}{\partial X} \tag{3I.2}$$

We know that $\frac{1}{(\mathcal{F}_1-\mathcal{F}_2)}$ is written as

$$\frac{1}{(\mathcal{F}_1-\mathcal{F}_2)} = \frac{1}{X\left[\frac{0.0046P_1+0.2}{24\Delta_1 S} - \frac{0.0046P_2+0.2}{24\Delta_2 S}\right]} = \frac{1}{cX} \tag{3I.3}$$

where c is just a constant. Therefore,

$$\begin{aligned}\frac{\partial\frac{1}{(\mathcal{F}_1-\mathcal{F}_2)}}{\partial X} &= -\frac{1}{cX^2} = -\frac{1}{\left[\frac{0.0046P_1+0.2}{24\Delta_1 S} - \frac{0.0046P_2+0.2}{24\Delta_2 S}\right]X^2} \\ &= -\frac{1}{(\mathcal{F}_1-\mathcal{F}_2)X}\end{aligned} \tag{3I.4}$$

It can be inferred from Equation (3I.4) that $\frac{\partial\frac{1}{(\mathcal{F}_1-\mathcal{F}_2)}}{\partial X} < 0$. Let's write

$$\frac{\mathcal{F}_1\mathcal{L}_1-\mathcal{F}_2\mathcal{L}_2}{\mathcal{F}_1-\mathcal{F}_2} = \frac{X\left(\frac{0.0046P_1+0.2}{24\Delta_1 S}\mathcal{L}_1 - \frac{0.0046P_2+0.2}{24\Delta_2 S}\mathcal{L}_2\right)}{X\left(\frac{0.0046P_1+0.2}{24\Delta_1 S} - \frac{0.0046P_2+0.2}{24\Delta_2 S}\right)} \tag{3I.5}$$

Hence, $\frac{\mathcal{F}_1\mathcal{L}_1-\mathcal{F}_2\mathcal{L}_2}{\mathcal{F}_1-\mathcal{F}_2}$ is independent of X and $\frac{\partial\frac{\mathcal{F}_1\mathcal{L}_1-\mathcal{F}_2\mathcal{L}_2}{\mathcal{F}_1-\mathcal{F}_2}}{\partial X} = 0$. Therefore,

$$\frac{\partial D^*}{\partial X} = \frac{I\beta_1(\rho-\alpha)}{(\beta_1-1)C}\frac{\partial\frac{1}{(\mathcal{F}_1-\mathcal{F}_2)}}{\partial X} < 0 \tag{3I.6}$$

CHAPTER 4. A NEW LATTICE METHOD FOR JUMP-DIFFUSION PROCESS APPLIED TO TRANSMISSION EXPANSION INVESTMENTS UNDER DEMAND AND DISTRIBUTED GENERATION (DG) UNCERTAINTIES

Introduction

After deregulation in electricity market in U.S., decision makers of transmission companies (we will use decision maker and transmission owner interchangeably) face critical uncertainties when they make investments because they do not have a prior information regarding decisions made by generation and distribution companies as well as communities. Demand for electricity is one of severe uncertainties because it continuously fluctuates even in a very small time interval (see, e.g., U.S. Department of Energy 2016). In addition, DGs have been installed in recent years with various sizes ranging from a couple of megawatts to tens of megawatts to meet local demand of electricity (see U.S. Energy Information Administration 2017 for a summary data listing various DG technologies preferred by utilities and societies as well as capacities installed in each year from 2006 to 2015).

Transmission investments (by transmission investments, we mean expansion investments) should be planned more strategically if DG is an alternative way to meet local demand. Professionals in electricity markets have already initiated discussions to evaluate the impact of DGs on costs and benefits of transmission investments. It is stated that transmission investments could be made more strategically if the rate of future adoption of DGs is estimated correctly (Biddle et al. 2014). Transmission investments might be delayed if DGs are installed because they meet a portion of local demand. It is a crucial uncertainty for transmission owners because they do not have prior information of DG installations. The research question arises as to in which way DG uncertainties (by DG uncertainty, we mean both installation and removal uncertainties of DGs) have effects on existing transmission network and future transmission investments. Our purpose in this study is to show how the value of a transmission investment is quantified under demand and DG uncertainties and to answer the research question by using real options approach. We assume that transmission owners use hybrid merchant/regulated investment approach, meaning that they have strategic flexibilities in making decisions such as delaying investments (see Pringles et al. 2014 for an example of work which adopt hybrid merchant/regulated investment approach for transmission expansion investments).

In literature, there exist some studies researching transmission investments with DG installation in transmission networks. However, rather than considering it as an uncertainty, they accept DG is a tool to acquire flexibility when decision makers attempt to expand the network. For instance, Buzarquis et al. (2010) quantify the value of deferring option gained by installing DGs from the point of view of distribution network owners. Luo et al. (2014) reveal how effective DGs are to defer transmission investments for a case study in Australia (see also Gil and Joos 2006; Piccolo and Siano 2009; Zhao et al. 2011). We emphasize that our study distinguishes itself from literature because we assume DG is an uncertainty for transmission owners.

Investment evaluation problems modeled with real options methodology can be solved with three different approaches. One can build an analytical model in order to obtain closed-form solutions. This approach is worthwhile as results and managerial insights do not depend on numerical values of parameters. However, handling with an analytically tractable model often requires to make many unrealistic and restrictive assumptions. Monte Carlo simulations have been proposed as an alternative especially for evaluating American options. It gives researchers great number of modeling flexibilities such as the ability of handling with jump and diffusion processes without enforcing a sequence between them. One the other hand, Monte Carlo simulation has a significant drawback from computational perspective. For instance, Longstaff and Schwartz

(2001) run 50,000 paths to obtain an average value of American options for each of different stochastic processes that they were interested in modeling of underlying stock prices. Lastly, lattice methods can be adopted because it is often said in the literature that they are able to save a great deal of computation time when compared to Monte Carlo simulation and get more accurate results (Areal et al. 2008). With lattice methods, however, one may not be sure about stability of results and managerial insights because they depend on (sensitive to) the numerical values of the set of parameters.

This chapter is structured as follows: In the following section, we show how a lattice model is constructed combining GBM and compound Poisson processes and we present a new lattice model, which requires much less computation time. We also present how we quantify the value of transmission investments. After that, we demonstrate our framework on a hypothetical example. The last section concludes the chapter by summarizing key points of the study and important managerial insights. Technical details of our framework are presented in appendices.

Mathematical Model

As stated before, transmission owners encounter demand and DG uncertainties when they invest. In literature for transmission investments planning, there exist several studies which assume that demand growth fits to GBM process (see, e.g., Loureiro et al. 2015 and Pringles et al. 2014). Besides those, there are also studies which statistically verify that real demand data fit to GBM process (see, e.g., Marathe and Ryan 2005). Since there are likely many consumption centers in a transmission network, we assume that each has demand growth modeled with GBM. Since installation or removal of a DG in a consumption center changes demand for electricity met by transmission lines, smooth path of demand (an infinitesimal change in an infinitesimal time interval, GBM) may abnormally jump to a higher or a lower level (a larger randomly occurring change). Since DG capacities are random as well, we make an assumption that DG uncertainties can be modeled with compound Poisson process. It is a tradition in the literature that these types of events, which happen rarely (that is why they are sometimes called rare events) are modeled with jump processes with the assumption that arrivals of events fit to Poisson arrival process (see, e.g., Martzoukos and Trigeorgis 2002).

We take advantage of lattice framework to model demand growth because finding a closed-form solution is impractical. There exist studies in financial option pricing literature which device lattice counterparts of jump-diffusion process (the process incorporating GBM and compound Poisson processes). Among others, we build our framework on the lattice model proposed by Hilliard and Schwartz (2005), with an extension on it. The authors propose discretization of jump and diffusion processes on two separate grids. They have matched the local moments of jump process with discrete branches. Note that the model proposed by Hilliard and Schwartz (2005) adopts one diffusion process and we extend it to multiple diffusion processes as each consumption center in a transmission network has its own demand modeled with GBM with different parameters (see also Amin 1993, Martzoukos and Trigeorgis 2002 and Dai et al. 2010 for other lattice models for jump-diffusion process). We also propose a new lattice model, which saves a great deal of computation time.

In this study, we focus on the following scenario: Suppose that there is a transmission network with centers (let N denote the set of centers in the network, and let N_D and N_G denote the set of consumption and generation centers, respectively) and power lines between centers (let M denote the set of power lines). Since existing power lines will not likely have sufficient capacity to meet future demand, the decision maker intends to expand the transmission network by installing power lines. However, he/she faces demand and DG uncertainties in consumption centers.

In the next sections, we first elaborate the way of lattice construction with a single diffusion process proposed by Hilliard and Schwartz (2005). Then, we explain how to combine multiple diffusion processes and their jumps in a lattice model. After that, we present how we reduce the computational complexity of the lattice model. At the end, we elaborate how to quantify the value of transmission investments.

Lattice Model of Jump-Diffusion Process for a Single Consumption Center

Hilliard and Schwartz (2005) give the risk-neutral form of jump-diffusion process as

$$\frac{dD_t}{D_t} = (r - \lambda\hat{\kappa})dt + \sigma dz_t + (\kappa - 1)ds_t \tag{4.1}$$

where D_t (MW) is demand at time point t, r (%/unit time) is risk-free interest rate, σ (%/unit time) is volatility of demand evolution, dz_t is the increment of Wiener process (i.e., $dz_t = \epsilon\sqrt{dt}$ where $\epsilon \sim N(0,1)$), and ds_t is the increment of jump process. If a jump occurs, ds_t takes value of 1; otherwise it is equal to 0. The number of DG events (installations or removals) are controlled by compound Poisson process with arrival rate λ (the number of events per unit time). κ (%) denotes jump magnitude defined as percentage change in D_t if a jump occurs. κ is generally assumed log-normally distributed with parameters (γ, δ^2) because the model is particularly tractable in this case (see, e.g., Merton 1976). $\hat{\kappa} = \mathrm{E}[\kappa] - 1$ where $\mathrm{E}[\kappa]$ is the expected value of κ, which is equal to $e^{\gamma + \frac{1}{2}\delta^2}$. It is further assumed that jump process is independent of diffusion process. For more explanations regarding Equation (4.1), please see Appendix 4.A.

Solution of Equation (4.1) is given as (see Appendix 4.B for details and Appendices 4.C and 4.D for supporting materials)

$$D_t = D_0 e^{\left(r-\lambda\hat{\kappa}-\frac{\sigma^2}{2}\right)t+\sigma z_t} \prod_{n=1}^{s_t} \kappa_n \tag{4.2}$$

Hilliard and Schwartz (2005) handle with Equation (4.2) by discretizing jump and diffusion processes on separate grids (bivariate tree). Equation (4.2) can be written as

$$\mathcal{D}_t = \ln\left(\frac{D_t}{D_0}\right) = X_t + Y_t \tag{4.3}$$

where $X_t = \left(r - \lambda\hat{\kappa} - \frac{\sigma^2}{2}\right)t + \sigma z_t$ and $Y_t = \sum_{n=1}^{s_t} \ln \kappa_n$. In the bivariate tree, both X_t and Y_t are normally distributed. Then,

$$\mathcal{D}_{t+1}^{+} = \mathcal{D}_t + \sigma\sqrt{\Delta t} + bh \tag{4.4}$$

$$\mathcal{D}_{t+1}^{-} = \mathcal{D}_t - \sigma\sqrt{\Delta t} + bh \tag{4.5}$$

$\mathcal{D}_t$ reaches the levels of $\mathcal{D}_{t+1}^{+}$ or $\mathcal{D}_{t+1}^{-}$ at the end of Δt time interval. Interested readers can check Figure 4.1, which shows the evolution of demand process represented in Equations (4.4) and (4.5). Note that we show diffusion and jump processes separately in Figure 4.1 although it is not a requirement. We draw them separately for expositional convenience as well as due to the fact that we do not know which process moves first. Note also that we use multiplicative model in Figure 4.1 instead of additive model of Equations (4.4) and (4.5) because we desire to illustrate the evolution of demand, not its natural logarithm. Δt is the length of a period in the lattice (period t is defined as from time point t to time point $t + 1$). $\pm\sigma\sqrt{\Delta t}$ represents up and down movements of diffusion process in the conventional binomial lattice, proposed by Cox et al. (1979). For jump process, b takes values on $\{0, \pm 1, \pm 2, \dots, \pm m\}$, meaning that the process is discretized on $2m + 1$ points. Jump process is typically map onto an odd number of points because middle node represents the case of no jump. Remaining jump nodes are symmetrical around the center node. The difference between successive jump nodes in the vertical order is denoted by h and it is expressed as $h = \alpha\sqrt{\gamma^2 + \delta^2}$ (α is a scale parameter. Hilliard and Schwartz (2005) state that the best simulation results are obtained when $\alpha = 1$). The risk-neutral probability of up movement of diffusion process ($+\sigma\sqrt{\Delta t}$) is given as

$$p = \frac{1}{2} + \frac{1}{2}\left(\frac{r - \lambda\hat{\kappa} - \frac{\sigma^2}{2}}{\sigma}\right)\sqrt{\Delta t} \tag{4.6}$$

Probabilities of jump branches, denoted by $q(b)$, are found by matching $2m$ moments of jump process. In other words,

$$\sum_{b=-m}^{m} (bh)^g \, q(b) = E\left[\left(\sum_{n=0}^{s_{\Delta t}} \ln \kappa_n\right)^g\right] = \mu_g \tag{4.7}$$

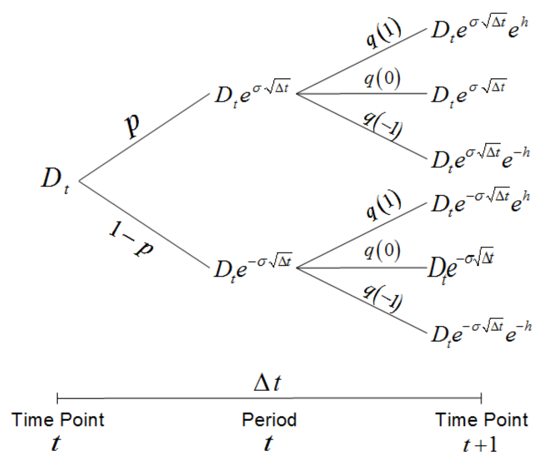

Figure 4.1 *Demand evolution lattice for m=1*

where μ_g is g^{th} moment of jump process and $g = \{0,1, \ldots, 2m\}$. Equation (4.7) is simplified as

$$\begin{bmatrix} q(-m) \\ q(-(m-1)) \\ \vdots \\ q(0) \\ \vdots \\ q(m-1) \\ q(m) \end{bmatrix} = \begin{bmatrix} 1 & 1 & \cdots & 1 & \cdots & 1 & 1 \\ (-m)^1 & (-(m-1))^1 & \cdots & 0 & \cdots & (m-1)^1 & m^1 \\ (-m)^2 & (-(m-1))^2 & \cdots & 0 & \cdots & (m-1)^2 & m^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ (-m)^{2m} & (-(m-1))^{2m} & \cdots & 0 & \cdots & (m-1)^{2m} & m^{2m} \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ \mu_1/h \\ \mu_2/h^2 \\ \vdots \\ \mu_{2m}/h^{2m} \end{bmatrix} \tag{4.8}$$

Finding μ_g for $g \geq 1$ is not straightforward. Hence, it is stated that when Δt is sufficiently small, μ_g can be approximated by cumulants $\mathcal{K}_g$; that is, $\mu_g \cong \mathcal{K}_g$ (See Appendix 4.E for details).

Hilliard and Schwartz (2005) propose discretization of jump-diffusion process with fixed jump magnitude as well. In this case, $\ln \kappa$ is a constant being equal to γ ($\delta = 0$, $\kappa = e^\gamma$, $\hat{\kappa} = e^\gamma - 1$, and $h = \gamma$ since $\delta = 0$). Equations (4.4) and (4.5) are adjusted by neglecting b (because there is one jump branch) and by replacing h with γ (see Figure 4.2). That is,

$$D_{t+1} = D_t e^{\sigma\sqrt{\Delta t}} e^{\gamma} \tag{4.9}$$

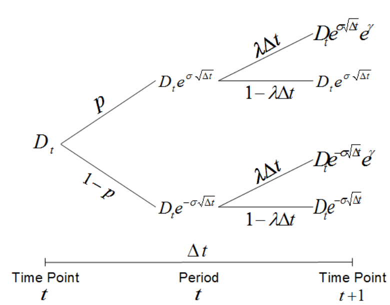

Figure 4.2 *Demand evolution lattice for fixed jump size*

Lattice Model of Jump-Diffusion Process for Multiple Consumption Centers

In literature, there exist many studies which put effort to discretize multiple diffusion processes by lattices. We build our framework on lattice model proposed by Wang and Min (2006). They create a lattice framework modeling the evolution of multiple diffusion processes to evaluate interrelated power generation projects.

Let i denote a consumption center in a transmission network and let D_t^i denote demand of this center at time t. D_t^i evolves following Equation (4.1) with parameters σ_i, κ_i, $\hat{\kappa}_i$, and λ_i. Probabilities of branches when it is discretized, p_i and $q_i(\cdot)$, have the same expressions given in Equations (4.6) and (4.8) with parameters h_i and μ_i.

Since there exist $|N_D|$ consumption centers in the network, diffusion part of demand lattice turns into a $2^{|N_D|}$-branch lattice. Wang and Min (2006) show joint risk-neutral probability of an arbitrary branch l of diffusion process as

$$p_l = \prod_{i=1}^{|N_D|} p_i' + \frac{1}{2^{|N_D|}} \sum_{i=1}^{|N_D|} \sum_{j=i+1}^{|N_D|} y_{ij}\rho_{ij} \tag{4.10}$$

where ρ_{ij} is correlation coefficient between D_t^i and D_t^j and $l = 1,2,\dots,2^{|N_D|}$. p_i' and y_{ij} are given as

$$p_i' = \begin{cases} p_i, & \text{if process } i \text{ moves upward in branch } l \\ 1 - p_i, & \text{if process } i \text{ moves downward in branch } l \end{cases} \tag{4.11}$$

$$y_{ij} = \begin{cases} 1, & \text{if processes } i \text{ and } j \text{ move in the same direction in branch } l \\ -1, & \text{if processes } i \text{ and } j \text{ move in the opposite direction in branch } l \end{cases} \tag{4.12}$$

If there exist two diffusion processes, demands D_t^i and D_t^j turn out to be $(D_t^i e^{\sigma_i\sqrt{\Delta t}}, D_t^j e^{\sigma_j\sqrt{\Delta t}})$, $(D_t^i e^{\sigma_i\sqrt{\Delta t}}, D_t^j e^{-\sigma_j\sqrt{\Delta t}})$, $(D_t^i e^{-\sigma_i\sqrt{\Delta t}}, D_t^j e^{\sigma_j\sqrt{\Delta t}})$, and $(D_t^i e^{-\sigma_i\sqrt{\Delta t}}, D_t^j e^{-\sigma_j\sqrt{\Delta t}})$

at the end of a period. If D_t^i and D_t^j are not correlated, risk-neutral probabilities of branches are the multiplication of individual probabilities; $p_i p_j$, $p_i(1-p_j)$, $(1-p_i)p_j$, and $(1-p_i)(1-p_j)$. Otherwise, these probabilities are written as $p_i p_j + \rho_{ij}/4$, $p_i(1-p_j) - \rho_{ij}/4$, $(1-p_i)p_j - \rho_{ij}/4$, and $(1-p_i)(1-p_j) + \rho_{ij}/4$ to take into account correlation.

A branch for diffusion process, which models the evolution of demand growth in multiple consumption centers, is followed by branches of jump processes, each of which pertains to a demand growth in a single consumption center. Since jump events are assumed to be independent, branch probabilities of jump processes are multiplication of individual jump probabilities. Each diffusion branch is followed by $(2m+1)^{|N_D|}$ jump branches (see Figure 4.3 for random jump magnitude and Figure 4.4 for fixed jump magnitude).

We denote a vector of demands in the lattice with $E_{(t,k)}$ where t denotes time points and k denotes states of the lattice. Value of k starts from 1 from the uppermost node and increments by 1 through the bottommost node for each t. We use $\wp$ to denote joint probabilities of diffusion and jump branches. For instance, in Figure 4.3, probability of $\left(D_t^i, D_t^j\right)$ to reach $\left(D_t^i e^{\sigma_i\sqrt{\Delta t}} e^{h_i}, D_t^j e^{-\sigma_j\sqrt{\Delta t}} e^{h_j}\right)$ is $\wp = (p_i p_j + \rho/4) q_i(1) q_j(1)$. In Figure 4.3 and Figure 4.4, we only show the jump branches emanating from the second diffusion branch for the sake of expositional convenience.

A New Lattice Model Reducing Computational Complexity

Note that above-proposed lattice model is computationally expensive. For even with two consumption centers and fixed jump magnitude (as in Figure 4.4), the number of nodes after one period turns out to be 16. It is obvious that the number of nodes after a large number of periods leads to a situation which cannot be managed from computational perspective. Therefore, we propose the following idea of reducing computational complexity of the model. Instead of allowing jump events (drawing jump branches) to happen at every period, we can let them happen at every v periods. We claim that given a small value of λ (let's define it as the average number of events per year), probability distributions of the terminal nodes obtained with the lattice model previously described and with this improvement idea approximate each other. Approximating probability distributions of the terminal nodes is a common approach in the literature. For instance, binomial lattice of Cox et al. (1979) proves that probability distribution of terminal nodes approximates the corresponding binomial distribution (see, e.g., Cudina 2018).

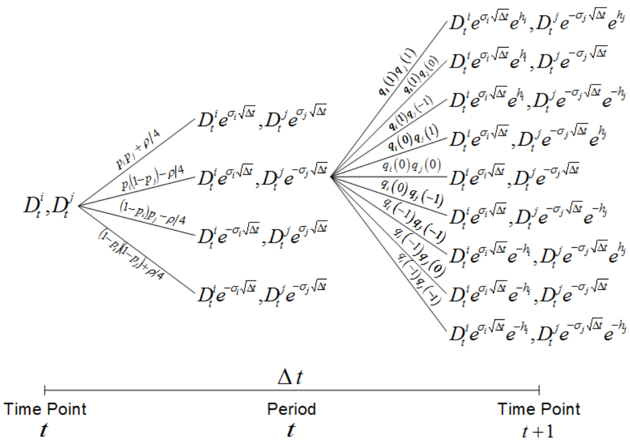

Figure 4.3 *Demand evolution lattice for two consumption centers with* $m = 1$

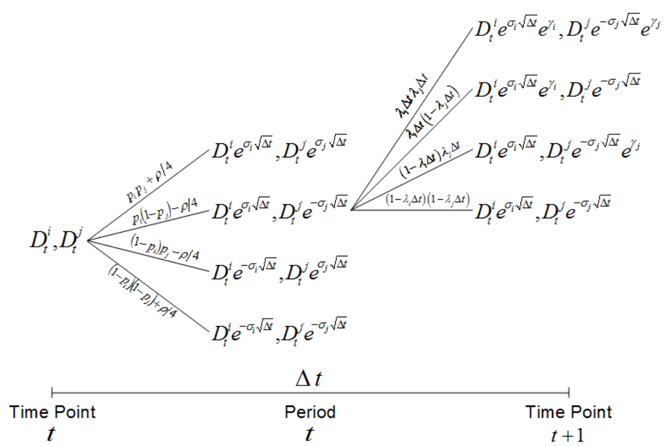

Figure 4.4 *Demand evolution lattice for two consumption centers (fixed jump)*

To show how the approximation works, let's consider two lattice models: One (say, model 1) allows jump event to occur every time period (as described in the preceding section) and the other one (say, model 2) allows jump event to happen at every v periods. Let's only focus on the initial parts of both lattice models (by initial parts, we mean lattice models starting with time point 0 and spanning through time point $v+1$). If it is shown that probability distributions of the terminal nodes of both partial lattice models approximate to each other, we do not need to pay attention to the rest of the models (after time point $v+1$) because a new partial lattice, which has the same structure starts at time point $v+1$.

For simplicity, we consider a single diffusion process and a fixed jump magnitude. In model 1, demand values at the terminal nodes (at time point $v+1$) have a general expression of $D_0\left(e^{\sigma\sqrt{\Delta t}}\right)^u\left(e^{-\sigma\sqrt{\Delta t}}\right)^d(e^{\gamma})^x$, meaning that there exist u times up movement and d times down movement of diffusion process as well as jump event happens x times in v periods. The probability of this node can be calculated as $\binom{v}{u}\binom{v}{x}p^u(1-p)^d(\lambda/\mathtt{m})^x(1-\lambda/\mathtt{m})^{v-x}$ where $\mathtt{m}$ is the number of periods in a year in the lattice model. With this model, we have the following observations:

If $x \geq 2$, $(\lambda/\mathtt{m})^x \to 0$. Hence, whole probability expression defined above approaches to 0.

If $x=1$, which means only one jump event happens in v periods, the probability expression turns into $\binom{v}{u}vp^u(1-p)^d(\lambda/\mathtt{m})(1-\lambda/\mathtt{m})^{v-1}$. Note that λ is small enough, $(1-\lambda/\mathtt{m})^{v-1} \to 1$, and thus the probability expression can be rewritten as $\binom{v}{u}p^u(1-p)^d(\lambda/\mathtt{m})v$.

If $x = 0$, which means no jump event happens in v periods, the probability expression turns into $\binom{v}{u}p^u(1-p)^d(1-\lambda/\mathtt{m})^v$. Notice again that λ is small enough, $(1-\lambda/\mathtt{m})^v \to 1$, and thus the probability expression approaches to $\binom{v}{u}p^u(1-p)^d$.

Let's consider model 2, which gives rise to a computational relaxation. Remember that in this model, there is no jump branch in periods prior to period v and there is a jump branch in period v. Note that up until period v, the lattice model is actually nothing more than well-known binomial lattice proposed by Cox et al. (1979). Therefore, probabilities of up and down movements of diffusion process do not consist of an arrival rate expression, λ. Since we assume that λ is sufficiently small, we are able to use probability expression given in Equation (4.6) by neglecting λ. Note that $\lambda = 0$ leads to risk-neutral probability of up movement as originally defined in Cox et al. (1979). With this consideration, demand values at the terminal nodes (at time point $v+1$) have general expressions of $D_0\left(e^{\sigma\sqrt{\Delta t}}\right)^u\left(e^{-\sigma\sqrt{\Delta t}}\right)^d e^{\gamma}$ and $D_0\left(e^{\sigma\sqrt{\Delta t}}\right)^u\left(e^{-\sigma\sqrt{\Delta t}}\right)^d$ depending on whether jump event occurs or not. The probabilities of these values are given as follows respectively:

$\binom{v}{u}p^u(1-p)^d(\lambda/\mathtt{m})v$, where $\lambda/\mathtt{m}$ is the probability of jump event to occur in a period and $(\lambda/\mathtt{m})v$ is the probability of jump event to occur in v periods.

$\binom{v}{u}p^u(1-p)^d(1-(\lambda/\mathtt{m})v)$. Notice that λ is small enough, we can write $(1-(\lambda/\mathtt{m})v) \to 1$, and the probability expression approaches to $\binom{v}{u}p^u(1-p)^d$.

Notice that the probability expression of model 1 when $x = 1$ approaches to the probability expression of model 2 when a jump event occurs at the last period. Similarly, the probability expression of model 1 when $x = 0$ approaches to the probability expression of model 2 when a jump event does not occur at the last period. Therefore, we claim that one can use model 2 instead of computationally expensive method as resulting probability distributions approach to each other and thus, resulting values of transmission networks will approach to each other. We remind that this relaxation works provided that the number of periods in the lattice models are sufficiently high and arrival rates of jump events are sufficiently small.

It is reasonable to accept model 2 in DG context because installation or removal of DGs are not events that happen frequently. It implies that arrival rates of these events are relatively small.

Quantification of Values of Transmission Investments

Hybrid merchant/regulated investment approach allows transmission owners to gain revenue from two major sources in the case of an expansion investment. Transmission owners gain from market participants such as distribution utilities and power generators (see, e.g., California ISO 2014a and 2014b for an example of transmission access charge in California). Owners additionally make money through Financial Transmission Rights, values of which are based on differences between LMPs in centers of the network. In light of this separation, we quantify values of transmission investments by modeling their revenues with LMP differences in the network (LMP-based revenues). In the case of an investment, we allow a supplementary revenue for transmission owners.

Our framework is conducted for each investment alternative (addition of a power line between two arbitrary centers) separately. We first consider the base case (the case that there is no investment in the network) and each demand vector in demand lattice is used to compute NPV of the network as state variable. Hence, a new lattice demonstrating the evolution of network value is created for the base case. We then proceed to evaluate each investment alternative. Since an investment can be postponed by the decision maker, we take into account different time points of investment (choices) separately. Choice t corresponds to the investment made at time point t for the selected investment alternative. For each choice, a different lattice showing the evolution of network value is created.

LMP is local price of electricity ($/MWh) and computed by solving OPF problem. LMP-based revenue of the network, denoted by R ($/hour), is calculated by

$$R = \sum_{i \in N_D} \pi_i D_i - \sum_{j \in N_G} \pi_j G_j \tag{4.13}$$

where π_i denotes LMP in center i and G_j (MW) denotes dispatched amount of generation center j at optimality of OPF problem. OPF problem is stated as

$$\min \sum_{i \in N_G} w_i G_i \tag{4.14}$$

$$G_i - D_i = \sum_{j \in N, j \neq i} \mathcal{B}_{ij}\left(\theta_i - \theta_j\right), \quad \forall i \in N \tag{4.15}$$

$$\mathcal{B}_{ij} = \begin{cases} -\mathscr{b}_{ij}, & \text{if } i \neq j \\ \sum\limits_{j \in N, i \neq j}^{N} \mathscr{b}_{ij}, & \text{otherwise} \end{cases} \tag{4.16}$$

$$-L_{ij} \leq \mathcal{B}_{ij}\left(\theta_i - \theta_j\right) \leq L_{ij}, \quad \forall (i,j) \in M \tag{4.17}$$

$$0 \leq G_i \leq \overline{G_\iota}, \quad \forall i \in N \tag{4.18}$$

where w_i ($/MWh) is generation cost of generation center i, $\mathcal{B}_{ij}$ is an element consisting of actual susceptance values (unit is Siemens; susceptance is defined as measure of easiness of power flow on a line), $\mathscr{b}_{ij}$ is susceptance value of the power line between centers i and j, θ_i is voltage angle in center i (unit is Radians; voltage angle is defined as an angle created by time shift in sinusoidal function of voltage), L_{ij} (MW) is capacity of the power line between centers i and j, and $\overline{G_i}$ (MW) is capacity of generation center i (For details of OPF problem, see, e.g., McCalley 2007 and Kocuk et al. 2016).

LMP in center i is obtained as follows. OPF problem is solved with given demand values and objective function value is recorded. Then, the problem is resolved with demand value increased by 1 MW in center i. The new objective function value minus its previously recorded value gives LMP in center i.

Note that the way we compute LMP is called layman's definition and it is practically used in electricity markets (California ISO 2005). LMP can be also calculated as shadow prices (value of Lagrange multipliers) of Equation (4.15) (see, e.g., Liu et al. 2009). However, these approaches may not give rise to the same set of values of LMPs. The reason is that shadow prices, by definition, are calculated with the infinitesimal change on demand values (Albouy 2018). It is obvious that an increase of 1 MW is not an infinitesimal change. In order to reconcile, we recalculate LMPs by using layman's definition, but with increasing demand values by a small amount such as 0.1 MW. We find that two sets of LMPs, calculated by layman's definition and Lagrange multipliers, are equal in this case.

Quantification of transmission network value for base case

Valuation starts with terminal nodes of the lattice. We assume that the network is removed at the end of modeling horizon ($t = T$) and this operation incurs a decommissioning cost. Hence, it implies that the value of the network at time point T is just the negative of decommissioning cost, denoted by C (\$). At time point $T - 1$, discounted total profit is calculated for Δt length of time (years or a fraction of a year) by making the assumptions that profit is realized at the end of each time period and demand does not change during Δt. In other words, discounted total profit gained in Δt duration is

$$P_{(T-1,k)} = \frac{8760\left(R_{(T-1,k)} - \mathbb{C}\right)\Delta t}{1+r} \tag{4.19}$$

where $R_{(T-1,k)}$ is network revenue calculated with Equation (4.13) and $\mathbb{C}$ (\$/hour) is operation and maintenance cost. NPV of the network at time point $T-1$ (\$) is finally defined as

$$V_{(T-1,k)} = P_{(T-1,k)} - \frac{\mathcal{C}}{1+r} \tag{4.20}$$

by taking into account the discounted decommissioning cost of the network. For the rest of intermediate nodes ($t < T-1$), discounted risk-neutral expected value of the successor nodes is added after calculating the profit with Equation (4.19). In other words,

$$V_{(t,k)} = P_{(t,k)} + \left(\sum_{\substack{l \in S_{(t,k)} \\ k \in S'_{(t,k)}}} \wp_l V_{(t+1,k)} \right) \Big/ 1+r \tag{4.21}$$

where $S_{(t,k)}$ denotes set of branches emanating from (t,k) and $S'_{(t,k)}$ denotes set of successor states of (t,k). $V_{(1,1)}$, obtained through recursive computation in Equation (4.21), is accepted as network value for base case.

Quantification of transmission network value with an investment

In the case of an investment, there are T choices for timing, and thus different NPV lattices are created for each by employing Equations (4.19), (4.20), and (4.21). When an investment is carried out at the beginning of period t, a supplementary revenue F (\$) and investment cost I (\$) are incorporated. If the investment is made at time point $T-1$, then

$$V_{(T-1,k)} = F - I + P_{(T-1,k)} - \frac{\mathcal{C}}{1+r} \tag{4.22}$$

If the investment is made at an arbitrary time point $t < T-1$, then

$$V_{(t,k)} = F - I + P_{(t,k)} + \left(\sum_{\substack{l \in S_{(t,k)} \\ k \in S'_{(t,k)}}} \wp_l V_{(t+1,k)} \right) \Big/ 1 + r \tag{4.23}$$

For Choice t, we calculate value of investment by subtracting $V_{(1,1)}$ (calculated for base case) from $V_{(1,1)}$ (calculated for the network with the investment made at time point t). If this difference is negative, value of investment is regarded as 0.

Numerical Example

In this section, the framework we develop is demonstrated on a simple numerical example. Let's assume that there exist three centers in the network (see Figure 4.5), each connected to another with a single power line. There are two generation centers (centers 1 and 2) and two consumption centers (centers 1 and 3). Parameters of generation centers and power lines are given in Figure 4.5. Initial demand values in centers 1 and 3 are 30 MW and 35 MW, respectively. We assume that susceptance of power lines are equal ($\mathscr{b}_{12} = \mathscr{b}_{13} = \mathscr{b}_{12} = 1$). We also assume that DGs have fixed sizes for the sake of simplification and they may be installed in consumption centers 1 and 3 with probabilities $\lambda_1 \Delta t = \lambda_3 \Delta t = 0.5$. Note that we just consider the installation of DGs, not their removals, to simply the problem in order to obtain fundamental managerial insights. Table 4.1 lists other hypothetical parameters of the numerical example. Note that in Table

4.1, whereas the first values for $\mathbb{C}$ and $\mathcal{C}$ represent base case, the second values are in place with an investment situation.

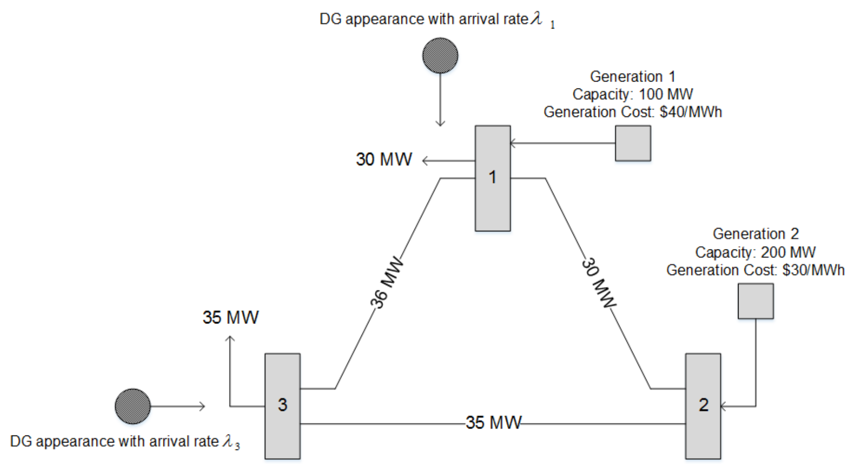

Figure 4.5 *A hypothetical three-center network*

Table 4.1 *Parameters of the numerical example*

Parameter	Value	Parameter	Value
σ_1	0.15/year	λ_1	0.5/year
σ_3	0.13/year	γ_1	-0.15
Δt	1 year	λ_3	0.5/year
T	2 years	γ_3	-0.15
r	0.05/year	ρ	0.1
$\mathbb{C}$	\$40/hour \$50/hour	$\mathcal{C}$	\$250,000 \$300,000
I	\$15,000,000	F	\$17,000,000

No Uncertainty Regarding DGs

In this section, we assume that there does not exist any uncertainty of DG installations or removals (see Figure 4.6. The numbers shown on branches are the risk-neutral probabilities). For investments, we assume that added power line has 4 MW capacity and it has the same susceptance as the existing power lines. Table 4.2 shows LMP-based revenues for different demand values. Throughout this numerical example, we use Matlab (fmincon function) to solve OPF problems and calculate the values of transmission networks.

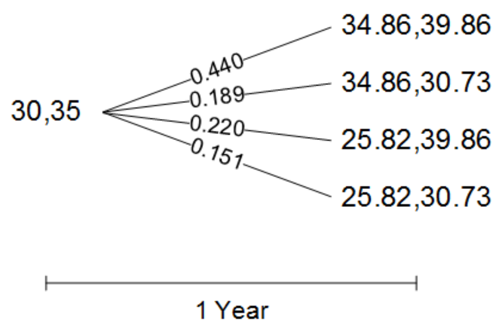

Figure 4.6 *Demand evolution lattice (DG uncertainty does not exist)*

Table 4.2 *LMP-based revenues, no uncertainty regarding DG*

(t,k)	$R_{(t,k)}$ for Base Case	$R_{(t,k)}$ for Investments Between Centers		
		1 and 2	1 and 3	2 and 3
(2,1)	964.19	425	500	975
(2,2)	450	425	500	251.98
(2,3)	964.19	425	500	975
(2,4)	0	0	0	0
(1,1)	450	425	500	975

Note that there exist two choices for timing of investments: At the beginning of the first year (Choice 1) and at the beginning of the second year (Choice 2). Table 4.3 lists $V_{(1,1)}$ values for base case and for investments with different investment times.

Table 4.3 *Values of investments, no uncertainty regarding DG*

Investments	Choices	$V_{(1,1)}$	Values of Investments
Base Case	-	\$8,608,074	-
Between Centers 1 and 2	1	\$7,326,242	0
	2	\$7,522,909	0
Between Centers 1 and 3	1	\$8,457,796	0
	2	\$8,028,844	0
Between Centers 2 and 3	1	\$14,539,154	\$5,931,080
	2	\$10,147,344	\$1,539,270

DG in Consumption Center 1

In this section, we analyze the case that a DG has a chance to be installed in consumption center 1 (see Figure 4.7). In Figure 4.7, state variables (separated by comma) are demand values in centers 1 and 3, respectively. Whereas the numbers on the left-hand side branches represent risk-neutral probabilities with correlation taken into account, the numbers on the right-hand side branches are probabilities of DG installations or of no installation.

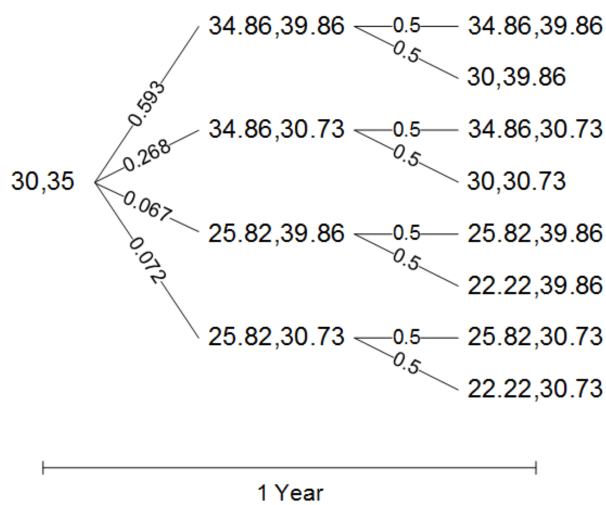

Figure 4.7 *Demand evolution lattice (DG in consumption center 1)*

Table 4.4 gives LMP-based revenues for different demand values. Table 4.5 lists $V_{(1,1)}$ values for base case and for investments with different choices.

DG in Consumption Center 3

In this section, we analyze the situation in which a DG installation may be realized in consumption center 3 (see Figure 4.8).

Table 4.6 reports LMP-based revenues for different demands shown in the lattice. $V_{(1,1)}$ values for base case and for investments with different timings are listed in Table 4.7.

Table 4.4 *LMP-based revenues, DG in consumption center 1*

(t,k)	$R_{(t,k)}$ for Base Case	$R_{(t,k)}$ for Investments Between Centers		
		1 and 2	1 and 3	2 and 3
(2,1)	964.19	425	500	975
(2,2)	964.19	425	500	975
(2,3)	450	425	500	251.98
(2,4)	450	425	500	0
(2,5)	964.19	425	500	975
(2,6)	0	204.22	0	975
(2,7)	0	0	0	0
(2,8)	0	0	0	0
(1,1)	450	425	500	975

Table 4.5 *Values of investments, DG in consumption center 1*

Investments	Choices	$V_{(1,1)}$	Values of Investments
Base Case	-	\$8,637,724	-
Between Centers 1 and 2	1	\$7,535,125	0
	2	\$7,731,792	0
Between Centers 1 and 3	1	\$8,640,544	\$2,820
	2	\$8,211,592	0
Between Centers 2 and 3	1	\$14,429,043	\$5,791,319
	2	\$10,037,233	\$1,399,509

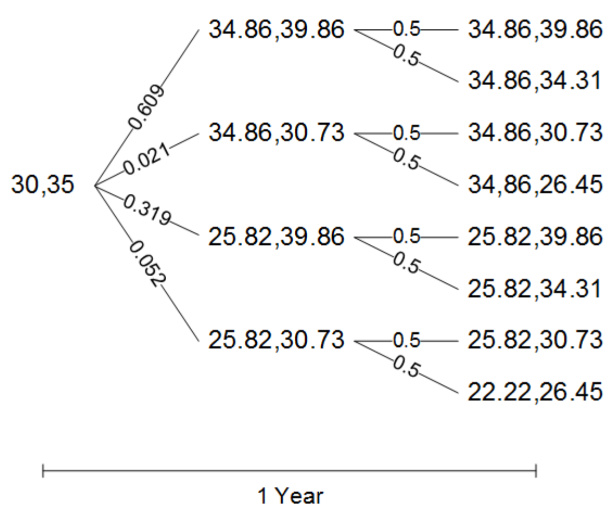

Figure 4.8 *Demand evolution lattice (DG in consumption center 3)*

Table 4.6 *LMP-based revenues, DG in consumption center 3*

(t, k)	$R_{(t,k)}$ for Base Case	$R_{(t,k)}$ for Investments Between Centers		
		1 and 2	1 and 3	2 and 3
(2,1)	964.19	425	500	975
(2,2)	450	425	500	975
(2,3)	450	425	500	251.98
(2,4)	450	425	500	0
(2,5)	964.19	425	500	975
(2,6)	0	425	0	0
(2,7)	0	0	0	0
(2,8)	0	0	0	0
(1,1)	450	425	500	975

Table 4.7 *Values of investments, DG in consumption center 3*

Investments	Choices	$V_{(1,1)}$	Values of Investments
Base Case	-	$7,589,197	-
Between Centers 1 and 2	1	$7,660,553	$71,356
	2	$7,857,219	$268,022
Between Centers 1 and 3	1	$8,215,456	$626,259
	2	$7,786,503	$197,306
Between Centers 2 and 3	1	$15,017,430	$7,428,233
	2	$10,625,621	$3,036,424

Discussions

$V_{(1,1)}$ values for base cases in three situations (no uncertainty regarding DGs, DG in consumption center 1 and DG in consumption center 3) lead to a significant managerial insight. It is observed that $V_{(1,1)}$ value computed with DG in consumption center 1 is not less than $V_{(1,1)}$ value computed with no uncertainty regarding DGs. It would be expected to see that installation of a DG most likely undervalues transmission lines because the community with a DG is partly in need of the lines. Our results contradict this expectation and emphasize that center 1 is not an 'influential' center to determine the dispatch amounts of generation centers. The reason is that whenever demand increases in this center, additional demand is met by its own generation plant. Center 3, on the other hand, does not have any generation plant and a demand increase highly impacts dispatch amounts of generation centers. Therefore, though a DG is installed and demand decreases in center 1, the network still produces high level of revenues because of high demand in center 3. This result is also supported by observation that $V_{(1,1)}$ value computed with DG in consumption center 3 is significantly less than $V_{(1,1)}$ value computed with no uncertainty regarding DGs. This discussion indicates decision makers should not always think that a DG decreases revenue gained by transmission lines. Instead, they should pay attention if the center in which a DG is installed is an influential center to determine dispatch amounts of generation centers.

It is also observed that the investment between centers 1 and 2 is delayed to the beginning of the second year. The new line decreases LMP-based revenues. Therefore, the decision maker intends to gain more revenue by not adding a power line at the beginning of the first year. In the cases that no uncertainty exists regarding DGs and a DG might be installed in consumption center 1, the investment is worthless at both time points. On the other hand, the investment is valuable in the case that a DG might be installed in consumption center 3. The fundamental reason is that $V_{(1,1)}$ value for base case with a DG installation in consumption center 3 is significantly less than other $V_{(1,1)}$ values for base cases. Hence, the investment capitalizes on lower $V_{(1,1)}$ value and makes profit.

Investment between centers 1 and 3 is made at the beginning of the first year as LMP-based revenues turn out to be higher throughout the first year when the investment is made. Similar to the investment between centers 1 and 2, the investment between centers 1 and 3 is worthless in the cases that there does not exist any uncertainty regarding DGs and a DG might be installed in consumption center 1. It is worth to make it if a DG might be installed in consumption center 3 because $V_{(1,1)}$ value for base case is lower and the decision maker capitalizes on it.

Investment between centers 2 and 3 is more profitable when it is made at the beginning of the first year because of higher LMP-based revenues resulting from the investment. Similar to other investments, the decision maker capitalizes on lower $V_{(1,1)}$ value when a DG might be installed in consumption center 3.

Conclusion

In this study, we propose a real options framework to quantify values of transmission investments under demand and DG uncertainties. We model the uncertain parameters with GBM and compound Poisson processes, and make use of lattice approach to discretize them. We propose an idea to reduce computational complexity stemming from combinations of jump and diffusion processes in a single lattice model. Key components of the proposed framework are demonstrated on a hypothetical numerical example based on three-center transmission network. The results indicate decision makers should not have a priori judgement that transmission network value decreases in the case a DG is installed. Instead, they should pay attention to locations of installations. If installation locations are not influential to determine dispatch amounts of generation centers, installations of DGs may not have effect on value of transmission lines. Future studies could involve two paths. First, correlation between GBM and compound Poisson processes could be taken into account because when demand for electricity increases, there may be higher chance of DG installations. Second, correlation between multiple compound Poisson processes could be considered because a community may prefer a DG if a neighbor community installs it due to the fact that they may have the same intention.

Appendix 4.A More Explanations about Equation (4.1)

Note that the coefficient of ds_t in Equation (4.1) is $\kappa - 1$ because the model has multiplicative property: In other words, when a jump occurs, $\frac{D_t - D_{t^-}}{D_{t^-}} = \kappa - 1$ where $D_{t^-} = \lim_{a \to t^-} D(a)$ denotes the demand just before a jump event occurs. Thus, $D_t = \kappa D_{t^-}$. Note that by restricting κ to be a positive value, we ensure that D_t never takes negative values.

Note also that the coefficient of dt in Equation (4.1) involves $-\lambda\hat{\kappa}$ because martingale property should be maintained. In other words, $(\kappa - 1)ds_t$ is regarded as an extra term, which increases or decreases the process. Therefore, it should be balanced with a component in the coefficient of dt. In other words,

$$\begin{aligned} E[(\kappa - 1)ds_t] &= E[\kappa - 1]E[ds_t] \\ &= \{E[\kappa] - 1\}E[ds_t] \\ &= \left\{e^{\gamma + \frac{1}{2}\delta^2} - 1\right\}\lambda dt \end{aligned} \tag{4A.1}$$

$E[ds_t]$ equals to λdt because ds_t is 1 with probability λdt and 0 with probability $1 - \lambda dt$. Therefore, it should be obvious that the term $-\lambda\hat{\kappa}$ should be added.

Appendix 4.B Solution of Equation (4.1)

In order to solve Equation (4.1), let's rewrite it as follows:

$$dD_t = D_t(r - \lambda\hat{\kappa})dt + D_t\sigma dz_t + D_t(\kappa - 1)ds_t \tag{4B.1}$$

Suppose $f(D_t) = \ln D_t$. Then $f'(D_t) = 1/D_t$ and $f''(D_t) = -1/D_t^2$. If we apply Ito's lemma for jump-diffusion process (see Appendices 4.C and 4.D), we get

$$\begin{aligned} d\ln D_t = \left[\frac{1}{D_t}D_t(r - \lambda\hat{\kappa}) - \frac{1}{2D_t^2}D_t^2\sigma^2\right]dt + \frac{1}{D_t}D_t\sigma dz_t \\ + [\ln(D_t + (\kappa - 1)D_t) - \ln D_t]ds_t \end{aligned} \tag{4B.2}$$

and

$$d\ln D_t = \left[r - \lambda\hat{\kappa} - \frac{\sigma^2}{2}\right]dt + \sigma dz_t + [\ln\kappa]ds_t \tag{4B.3}$$

If we integrate both sides,

$$\int_0^t d\ln D_a = \int_0^t \left(r - \lambda\hat{\kappa} - \frac{\sigma^2}{2}\right)da + \int_0^t \sigma dz_a + \int_0^t \ln\kappa\, ds_a \tag{4B.4}$$

and

$$\ln D_t - \ln D_0 = \left(r - \lambda\hat{\kappa} - \frac{\sigma^2}{2}\right)t + \sigma z_t + \sum_{n=1}^{s_t}\ln\kappa_n \tag{4B.5}$$

where we assume $z_0 = 0$. The last term follows from the fact that integral from 0 to t means the sum of the jump events. Therefore,

$$D_t = D_0 e^{\left(r - \lambda\hat{\kappa} - \frac{\sigma^2}{2}\right)t + \sigma z_t + \sum_{n=1}^{s_t}\ln\kappa_n} \tag{4B.6}$$

and

$$D_t = D_0 e^{\left(r - \lambda\hat{\kappa} - \frac{\sigma^2}{2}\right)t + \sigma z_t}\prod_{n=1}^{s_t}\kappa_n \tag{4B.7}$$

Appendix 4.C Ito's Lemma and GBM

Let z_t be a Brownian motion at time t. Then the instantaneous change in an arbitrary function f of z_t is calculated as (see, e.g., Klebaner 2005):

$$df(z_t) = f'(z_t)dz_t + \frac{1}{2}f''(z_t)dt \tag{4C.1}$$

Ito's lemma can also be written for a general Ito process. D_t is said to be an Ito process if

$$dD_t = m(D_t, t)dt + \sigma(D_t, t)dz_t \tag{4C.2}$$

where $m(D_t, t)$ and $\sigma(D_t, t)$ are drift and volatility parameters (Dixit and Pindyck 1994). Ito's lemma is given for an arbitrary function f as

$$df(D_t) = f'(D_t)dD_t + \frac{1}{2}f''(D_t)d[D, D]_t \tag{4C.3}$$

where $d[D, D]_t$ is the quadratic variation of Ito process, which is defined as:

$$d[D, D]_t = \int_0^t \sigma^2(D_a, a)da = \sigma^2(D_t, t) \tag{4C.4}$$

Therefore,

$$df(D_t) = f'(D_t)dD_t + \frac{1}{2}f''(D_t)\sigma^2(D_t, t)dt \tag{4C.5}$$

If we plug dD_t, we get

$$\begin{aligned} df(D_t) &= f'(D_t)[m(D_t, t)dt + \sigma(D_t, t)dz_t] \\ &\quad + \frac{1}{2}f''(D_t)\sigma^2(D_t, t)dt \\ &= \left[f'(D_t)m(D_t, t) + \frac{1}{2}f''(D_t)\sigma^2(D_t, t)\right]dt \\ &\quad + f'(D_t)\sigma(D_t, t)dz_t \end{aligned} \tag{4C.6}$$

Let's assume that D_t follows GBM; that is,

$$dD_t = mD_t dt + \sigma D_t dz_t \tag{4C.7}$$

Note that $m(D_t, t)$ and $\sigma(D_t, t)$ in a general Ito process take the form of mD_t and σD_t in GBM. Suppose $f(D_t) = \ln D_t$. Hence, $f'(D_t) = 1/D_t$ and $f''(D_t) = -1/D_t^2$, and

$$df(D_t) = \left[\frac{1}{D_t} mD_t - \frac{1}{2D_t^2}\sigma^2 D_t^2\right] dt + \frac{1}{D_t}\sigma D_t dz_t$$
$$= \left(m - \frac{1}{2}\sigma^2\right) dt + \sigma dz_t \qquad (4C.8)$$

If we integrate both sides,

$$\int_0^t d\ln D_a = \int_0^t \left(m - \frac{1}{2}\sigma^2\right) da + \int_0^t \sigma dz_a \qquad (4C.9)$$

and

$$D_t = D_0 e^{\left(m-\frac{1}{2}\sigma^2\right)t+\sigma z_t} \qquad (4C.10)$$

If we take the expectation of both sides, we get

$$E[D_t] = D_0 e^{\left(m-\frac{1}{2}\sigma^2\right)t} E[e^{\sigma z_t}] \qquad (4C.11)$$

It is known that $z_t \sim N(0, t)$ and $\sigma z_t \sim N(0, \sigma^2 t)$. Hence,

$$E[e^{\sigma z_t}] = e^{0+\frac{1}{2}\sigma^2 t} \qquad (4C.12)$$

Finally,

$$E[D_t] = D_0 e^{\left(m-\frac{1}{2}\sigma^2\right)t} e^{\frac{1}{2}\sigma^2 t} = D_0 e^{mt} \qquad (4C.13)$$

Risk-neutrality implies that expected value of the process at time t is equal to initial value. Moreover, when time-value is a significant factor, discounted value of the process should be taken into account. Thus, for the above process,

$$E[D_t]e^{-rt} = D_0 \qquad (4C.14)$$

should hold to maintain the martingale property, or risk-neutrality property. It can be concluded that if m is replaced with r in GBM, its risk-neutral form is obtained.

Appendix 4.D Jump Process and Ito's Lemma

Ito's lemma can be applied as well for jump-diffusion process (see, e.g., Birkbeck 2013). Let $\mathcal{N}_t$ be a Poisson counting process. Particularly,

$$d\mathcal{N}_t = \mathcal{N}_{t+dt} - \mathcal{N}_t \tag{4D.1}$$

In this process, $d\mathcal{N}_t$ takes non-negative integer values. Since it is Poisson distributed, we can write

$$P(d\mathcal{N}_t = \mathcal{k}) = e^{-\lambda dt}\frac{(\lambda dt)^{\mathcal{k}}}{\mathcal{k}!} \tag{4D.2}$$

Since dt is very small, probability approaches to 0 when $\mathcal{k} \geq 2$. Therefore, for 0 and 1, $P(d\mathcal{N}_t = 0) = e^{-\lambda dt}$ and $P(d\mathcal{N}_t = 1) = e^{-\lambda dt}\lambda dt$. If Taylor's expansion of the exponential is applied, we get

$$d\mathcal{N}_t = \begin{cases} 0, & \text{with probability } 1 - \lambda dt \\ 1, & \text{with probability } \lambda dt \end{cases} \tag{4D.3}$$

If we consider an arbitrary function f,

$$df(\mathcal{N}_t) = f(\mathcal{N}_{t+dt}) - f(\mathcal{N}_t) \tag{4D.4}$$

or

$$df(\mathcal{N}_t) = f(\mathcal{N}_t + d\mathcal{N}_t) - f(\mathcal{N}_t) \tag{4D.5}$$

Considering the probabilities of $d\mathcal{N}_t$, we write

$$df(\mathcal{N}_t) = \begin{cases} 0, & \text{with probability } 1 - \lambda dt \\ f(\mathcal{N}_t + 1) - f(\mathcal{N}_t), & \text{with probability } \lambda dt \end{cases} \tag{4D.6}$$

Since $df(\mathcal{N}_t)$ and $d\mathcal{N}_t$ have two consequences with the same probabilities, they are incorporated into a single equation as

$$df(\mathcal{N}_t) = [f(\mathcal{N}_t + 1) - f(\mathcal{N}_t)]d\mathcal{N}_t \tag{4D.7}$$

This process can be generalized to the random jump magnitude. Let $\mathcal{X}_t$ be a process jumping at the same time with Poisson counting process, but the magnitude of the jump is a random variable J_t. Therefore,

$$d\mathcal{X}_t = J_t d\mathcal{N}_t \tag{4D.8}$$

and,

$$df(\mathcal{X}_t) = \begin{cases} 0, & \text{with probability } 1 - \lambda dt \\ f(\mathcal{X}_t + J_t) - f(\mathcal{X}_t), & \text{with probability } \lambda dt \end{cases} \tag{4D.9}$$

It can be expressed in a single equation as

$$df(\mathcal{X}_t) = [f(\mathcal{X}_t + J_t) - f(\mathcal{X}_t)]d\mathcal{N}_t \tag{4D.10}$$

Up to this point, $d\mathcal{X}_t$ has had only jumps. In other words, the change in $d\mathcal{X}_t$ between two jumps has been 0. In jump-diffusion process, however, the change in $d\mathcal{X}_t$ between two jump events is different than 0 because of the effects of drift and volatility parameters. Mathematically speaking, $\mathcal{X}_t$ is said to follow jump-diffusion process if it has the following stochastic differential equation

$$d\mathcal{X}_t = m(\mathcal{X}_t, t)dt + \sigma(\mathcal{X}_t, t)dz_t + J_t d\mathcal{N}_t \tag{4D.11}$$

Ito's lemma for this process with an arbitrary function f is expressed as

$$\begin{aligned} df(\mathcal{X}_t) = {} & \left[f'(\mathcal{X}_t)m(\mathcal{X}_t, t) + \frac{1}{2}f''(\mathcal{X}_t)\sigma^2(\mathcal{X}_t, t)\right]dt \\ & + f'(\mathcal{X}_t)\sigma(\mathcal{X}_t, t)dz_t + [f(\mathcal{X}_t + J_t) - f(\mathcal{X}_t)]d\mathcal{N}_t \end{aligned} \tag{4D.12}$$

Appendix 4.E Calculating Cumulants

For a compound Poisson random variable $\mathcal{Y} = \sum_{i=1}^{\mathbb{N}} \mathbb{X}_i$ where $\mathbb{N}$ is Poisson distributed random variable with parameter $\lambda_{\mathbb{N}}$ and $\mathbb{X}_i$'s are independent and identically distributed random variables, the moment generation function $\mathcal{M}_{\mathcal{Y}}(t)$ is given as (see, e.g., Ma 2010)

$$\mathcal{M}_{\mathcal{Y}}(t) = e^{\lambda[\mathcal{M}_{\mathbb{X}}(t)-1]} \tag{4E.1}$$

Cumulant generating function of $\mathcal{Y}$ denoted by $\Psi_{\mathcal{Y}}(t)$ is calculated as the logarithm of the corresponding moment generating function. That is,

$$\Psi_{\mathcal{Y}}(t) = \lambda[\mathcal{M}_{\mathbb{X}}(t) - 1] \tag{4E.2}$$

In log-normal jump distribution, $\mathbb{X}$ is a normally distributed random variable. Thus, by using the fact that moment generating function of $\mathbb{X}$ is $e^{\eta t+\frac{\beta^2 t^2}{2}}$ where η and β are mean and standard deviation of $\mathbb{X}$, one can write

$$\Psi_{\mathcal{Y}}(t) = \lambda\left[e^{\eta t+\frac{\beta^2 t^2}{2}} - 1\right] \tag{4E.3}$$

Cumulants are calculated by taking the sequential derivatives of $\Psi_{\mathcal{Y}}(t)$ with respect to t and by setting t equal to zero. In other words, the first cumulant $\mathcal{K}_1$ is

$$\mathcal{K}_1 = \left.\frac{d\Psi_{\mathcal{Y}}(t)}{dt}\right|_{t=0} = \lambda\eta \tag{4E.4}$$

Similarly, $\mathcal{K}_2$ is

$$\mathcal{K}_2 = \left.\frac{d^2\Psi_{\mathcal{Y}}(t)}{dt^2}\right|_{t=0} = \lambda(\eta^2 + \beta^2) \tag{4E.5}$$

The rest of the cumulants can be computed in the same way. Note that we need the first $2m$ cumulants if we discretize the jump distribution with $2m + 1$ branches. Moreover, since we account for the number of events in Δt time interval, the parameter of compound Poisson process turns out to be $\lambda\Delta t$. That is why, λ is replaced with $\lambda\Delta t$ in the above calculations.

The reason why $\mu_i \approx \mathcal{K}_i$ when Δt is sufficiently small should also be given. For $i = 1$, one can say that $\mu_1 = \lambda\Delta t$. This result can be reached by summing up several normally distributed random variables and taking the expectation of the sum. For $i \geq 2$, it is known that $\mu_i = \mathcal{K}_i + O(\Delta t^2)$ where $O(\Delta t^2)$ includes the multiplication $\mathcal{K}_j$'s when $j < i$. Thus, $\Delta t \gg \Delta t^2, \Delta t^{\frac{3}{2}}, \Delta t^3$ ... if Δt is chosen sufficiently small. Therefore, it can be concluded that $\mu_i = \mathcal{K}_i$ for all $i \geq 1$ (see, e.g., Kendall 1945)

Switching from moment to cumulant in this context is useful because $\mathcal{M}_{\mathcal{Y}}(t)$ is a function of exponential to the power of another exponential. Therefore, taking the derivative of $\mathcal{M}_{\mathcal{Y}}(t)$ with respect to t becomes a tedious job. Instead, taking advantage of the cumulant (another distribution characteristic alternative to the moment) is very advantageous because of its logarithm property.

CHAPTER 5. OVERALL DISSERTATION APPENDIX

In this chapter, we address various issues raised during my prelim exam and our discussions with fellows.

Equality of Susceptance Values While Power-Carrying Capacities Differ

This subsection refers to an issue which appears in Chapter 2 and Chapter 4. In numerical examples of both of these chapters, we assume that existing power lines in the network have the same susceptance values, but their power-carrying capacities differ. We further assume that the power lines which will be installed in the network have the same susceptance values as existing power lines, but their capacities are again different.

We verify that Bushnell and Stoft (1995) make the same assumptions implicitly. They assume that the power lines, existing and to be added, share a common susceptance value.

Note that there exist two different capacity definitions. One is called thermal limit of a power line. It indicates that a power line has its own physical properties and if an excess amount of power flows on that line, it is likely be physically damaged. Therefore, power transmission companies (or other related bodies) set a maximum limit of power flow, which is generally less than thermal limit for security reasons. In this discussion, we mean thermal limit by power line capacities. We verify that thermal limit of a power line, in reality, is limited to so-called Surge Impedance Loading. It is stated in Power Delivery Consultants, Inc. (2013) that Surge Impedance Loading is the proportion of end bus voltage to characteristic impedance of a power line. It is further stated that characteristic impedances of sufficiently long power lines are approximately equal to each other, and thus, Surge Impedance Loading uniquely depends on end bus voltages.

To summarize, we verify that our assumptions regarding the equality of susceptance values do not harm the models.

Different Approaches to Calculation of LMPs

This subsection addresses different approaches used in calculating LMPs. There are two classical ways of obtaining LMPs in a given transmission network. First approach is using the values of Lagrange multipliers of the constraints in an OPF problem representing the power flow balance in centers. The second way (so-called layman's definition) adopts a re-optimization approach, which means that LMP of a given center, say i, is the difference between objective function values of OPF problem, solved with original demand value in center i and with a demand in this center increased by 1 MW (California ISO 2005). We adopt the second approach to calculate LMPs in Chapter 2 and Chapter 4 because it is a practical way utilized in California electricity market. Moreover, we think that it is more intuitive and easier to explain what LMP is and how it is calculated.

It is obvious that these approaches *may* give rise different sets of LMPs for a given set of demand values in a transmission network. The reason lies in the definition of a Lagrange multiplier. Lagrange multiplier of a constraint is the amount of change in the objective function value of an optimization problem when the right-hand side of the constraint is increased infinitesimally. It is clear that 1 MW increase in layman's definition is not infinitesimal. To observe it better, we conduct a simple study to compare LMPs obtained by two approaches and try to see if an infinitesimal change adopted in layman's definition would give rise to same LMPs obtained by Lagrange multipliers.

We refer to the demand values given in Table 2.2 and recalculate LMPs with two approaches. In Table 5.1, (i) indicates that LMPs are calculated by layman's definition, (ii) indicates that LMPs are calculated by layman's definition, but with 0.01 MW increases in nodal balance constraints, and (iii) indicates that LMPs are calculated as values of Lagrange multipliers of the nodal balance constraints. We remind that π_i denotes LMP in center i of the transmission network, taken as a case in numerical example of Chapter 2. Table 5.1 reflects that the only difference between results of layman's definition (i) and Lagrange multipliers (iii) is LMP in center 3 for a demand value of 52 MW. Note that when we implement layman's definition with 0.01 MW increase (ii), all sets of LMPs turn out to be the same.

Table 5.1 *Values of LMPs calculated with different approaches*

Demand value at center 3 (MW)	(i)			(ii)			(iii)		
	π_1	π_2	π_3	π_1	π_2	π_3	π_1	π_2	π_3
59.22	40	30	50	40	30	50	40	30	50
45.66	30	30	30	30	30	30	30	30	30
52	30	30	40	30	30	30	30	30	30

An Alternative Performance Measure (Based on Fuel Cost Saving) for Jumboization

In this section, we revisit Chapter 3 and try to solve jumboization investment problem by removing two critical assumptions made previously (Assumptions 3 and 4). We remind that the replenishment oiler makes a round-trip voyage between two constant locations (Assumption 3), and it moves at a constant speed during voyages (Assumption 4). In this section, we think that if the previous model functions with a constant distance restriction, then it should also work regardless of the magnitude of the distance. It implies that we can think of an infinitesimal distance between locations without enforcing a numerical value beforehand. Another change we adopt in this section is to consider fuel cost saving per unit demand, instead of per unit displacement because light ship materials (hull structure, permanent materials on the ship, etc.) comprise of an auxiliary system, which only exists because of the requirement of transporting fuel to U.S. Navy ships at sea.

It is stated in Chapter 3 that $0.0046P + 0.2$ is the maximum amount of daily consumption of bunker fuel for a ship given that P is the maximum power required to move the ship with full cargo. In this section, we directly calculate the amount of bunker fuel consumed by the ship per unit demand as $(0.0046P + 0.2)/D_t$ (we remind that D_t is the amount of fuel demanded by an U.S. Navy ship at sea) and fuel cost saving gained by jumboization as

$$\frac{0.0046P_1 + 0.2}{D_t} - \frac{0.0046P_2 + 0.2}{D_t} \tag{5.1}$$

where P_1 and P_2 denote the maximum power required before and after jumboization. Value of the project (in this case, value of the project is the lengthened form of the replenishment oiler) is calculated as the expected value of all future fuel cost savings discounted with factor, ρ. In other words, value of the project, $V(D_x)$, is

$$V(D_x) = E\left[\int_0^{\infty} \frac{0.0046(P_1 - P_2) + 0.2}{D_t} e^{-\rho t} dt\right] \tag{5.2}$$

where D_x is assumed to be the demand level at which jumboization is done (note that x does not denote a time point) and lower bound of integral represents the time of jumboization. Equation (5.2) is simplied as

$$V(D_x) = [0.0046(P_1 - P_2) + 0.2]E\left[\int_0^{\infty} \frac{1}{D_t} e^{-\rho t} dt\right] \tag{5.3}$$

In order to calculate expected value of the integral in Equation (5.3), we need to verify if we can change the order of expectation and integration operators. Fubini's theorem (Klebaner 2005) states that the change of order is viable if

$$\int_0^{\infty} E\left|\frac{1}{D_t}e^{-\rho t}\right| < \infty \tag{5.4}$$

Therefore, expected value of reciprocal of a GBM process should be known. We can write (by using the facts $\frac{1}{D_t}e^{-\rho t} > 0$ and $E\left[\frac{1}{D_t}e^{-\rho t}\right] = E\left[\frac{1}{D_t}\right]e^{-\rho t}$)

$$E\left[\frac{1}{D_t}\right] = E\left[\frac{1}{D_0 e^{\left(\alpha-\frac{\sigma^2}{2}\right)t} e^{\sigma z_t}}\right] \tag{5.5}$$

because we know the solution of D_t (see Appendix 3.D). Note that z_t is a Brownian increment. Hence, Equation (5.5) is simplified as

$$E\left[\frac{1}{D_t}\right] = \frac{1}{D_0 e^{\left(\alpha-\frac{\sigma^2}{2}\right)t}} E\left[e^{-\sigma z_t}\right] \tag{5.6}$$

It is known that negative of a GBM process is itself. Therefore, it can be simply claimed $e^{-\sigma z_t} = e^{\sigma z_t}$. We know that $E[e^{\sigma z_t}] = e^{\frac{\sigma^2 t}{2}}$ and thus,

$$E\left[\frac{1}{D_t}\right] = \frac{e^{\frac{\sigma^2 t}{2}}}{D_0 e^{\left(\alpha-\frac{\sigma^2}{2}\right)t}} \tag{5.7}$$

Therefore,

$$E\left[\frac{1}{D_t}\right] = \frac{e^{(\sigma^2-\alpha)t}}{D_0} \tag{5.8}$$

Since jumboization is done when demand is at the level of D_x, we can replace D_x with D_0 in Equation (5.8). If we plug Equation (5.8) into inequality (5.4), we get

$$\int_0^{\infty} \frac{e^{(\sigma^2-\alpha-\rho)t}}{D_x} < \infty \tag{5.9}$$

Integral part of inequality (5.9) is solved as

$$\int_0^{\infty} \frac{e^{(\sigma^2-\alpha-\rho)t}}{D_x} = \frac{1}{D_x(\alpha+\rho-\sigma^2)} \tag{5.10}$$

Because of our technical assumption $\alpha - \frac{\sigma^2}{2} > 0$ (see Equation (3.23)), we can write $\sigma^2 < 2\alpha$. Another technical assumption indicates $\rho > \alpha$ (see Equation (3.12)). Therefore, we say that $\alpha + \rho - \sigma^2 > 0$. It verifies that Fubini's theorem is applicable because $\frac{1}{D_x(\alpha+\rho-\sigma^2)} < \infty$. Hence, change of order of integration and expectation operators can be performed in Equation (5.3). Finally, Equation (5.3) is written as

$$V(D_x) = \frac{0.0046(P_1 - P_2) + 0.2}{D_x(\alpha+\rho-\sigma^2)} \tag{5.11}$$

By using the same value-matching and smooth-pasting conditions (Equations (3.19) and (3.20)), we can obtain

$$D^* = \frac{(\beta_1 - 1)[0.0046(P_1 - P_2) + 0.2]}{(\alpha+\rho-\sigma^2)\beta_1 I} \tag{5.12}$$

Note that $D^* > 0$ because $\beta_1 > 1$, $P_1 - P_2 > 0$ as jumboization leads to fuel cost saving resulting from less power required to move the ship, $\alpha + \rho - \sigma^2 > 0$ as shown above and $I > 0$ as obvious.

Equation (5.12) is inherently an interesting finding because jumboization cost, which may be in the order 10 million dollars is in the denominator. It shows that D^* is a positive number, but very close to zero. Although it does not seem to be a valuable finding, it is not surprising and it is in line of the expectation because underlying performance measure is fuel cost saving per unit demand. If demand is high, fuel cost saving per unit demand will be naturally low. Hence, threshold demand level to jumboize the ship is very close to zero to maximize fuel cost saving per unit demand.

Note that two sets of parameters in Equation (5.12) have units depending on two different time intervals. Whereas α, ρ, and σ are related to time intervals for demand realizations, P_1 and P_2 are related to time intervals for voyages of the replenishment oiler. For instance, if demand by the receiving ship is realized at every two weeks, units of α, ρ, and σ should be percent per two weeks. On the other hand, it does not necessarily mean that the replenishment oiler will spend whole two weeks in voyages. It can carry the fuel to the receiving ship and go back to its original port in, say, two days. In this case, $0.0046(P_1 - P_2) + 0.2$ should be multiplied with two because $0.0046(P_1 - P_2) + 0.2$ has unit of gallon per day. As a summary, one should be cautious in using Equation (5.12) because of various units depending on different time intervals.

Alternative Performance Measures for Jumboization

In this section, we list alternative performance measures other than fuel cost saving. Since jumboization in the U.S. Navy is a type of non-profit investments, there must be other performance measures or motivations for the decision makers in the U.S. Navy who decide on jumboization. We need to state that we have an extreme lack of historical facts about jumboization in the U.S. Navy. All we know is that a few of replenishment oilers were jumboized at different times in the past. However, we do not know how they decided (we know that they wanted to increase the capacity of the ships, but nothing more than this) and we are not aware of financial aspects (investment cost, etc.) of all these operations. Therefore, we draw inferences whenever we encounter a narrative regarding jumboization investments in the U.S. Navy.

First of all, we admit the performance measure based on fuel cost saving sounds like it is a secondary objective of jumboization investments, or a natural output of these investments. As stated previously, the decrease in wave-making resistance leads to less power required to move the ship with the same amount of cargo and thus, less fuel amount consumed by the ships. However, it would not be completely correct to think that the decision makers in the U.S. Navy jumboized the replenishment oilers just for the sake of fuel cost saving. We adopt this performance measure in Chapter 3 because fuel cost saving is the unique and clearest measure that can be converted to monetary values.

However, we later on come across a website (Finnlines 2017) which introduces jumboization operations in a transportation company located in Finland. According to it, Finnlines jumboizes its four large vessels in 2017 in order to reduce energy consumption per unit transported cargo. It strengthens our idea that fuel cost saving can be taken into account as a performance measure for jumboization investments.

We can also list other performance measures different than fuel cost saving. IT1me (2015) indicates that the U.S. Navy jumboized eight of oilers to increase their individual capacities to 180,000 barrels. The decision makers in the U.S. Navy considered that this amount would be sufficient to support a supercarrier and its jet air wing's fuel needs. Furthermore, Wildenberg (1996) states that the U.S. Navy had only a few oilers which had large enough capacities to fill an empty fast combat ship in 1960s. They were motivated by this need and they jumboized five of oilers. We interpret these narratives in the way that if jumboization was not in place, the replenishment oilers would have to travel more frequently to meet the fuel demand. Therefore, by lengthening of replenishment oilers, the number of voyages required and the total amount of time spent in travels would be decreased. Hence, time saving could be an important performance measure for jumboization in the U.S. Navy. We find a similar study which considers improvement in time saving as a performance measure in infrastructure context (McConnell 2007). It states that there is a special (managed) line on Katy Freeway in Houston, Texas. This line used to be a HOV2+, which means vehicles with 2 passengers or more can use it. Later on, in order to increase the capacity of Katy Freeway (capacity of road is basically measured by the number of vehicles passing a point in an hour under normal road and traffic conditions), this line was converted to HOT3+ under Quick Ride program in 2007. It implies that managed line started being used by vehicles with 3 passengers or more, but vehicles with 2 passengers was again able to use it by paying a $2 fee. In this way, the capacity of Katy Freeway was increased and an amount of fund was collected. However, the performance of this program was measured with time saving per passenger. It turned out that Quick Ride program was able to give rise 14 miles per hour larger speed on average. In order to convert this measure to a monetary unit, the reduction in fuel consumption of the cars were taken into account based on the reduction in time spent on the road and the increase in speed. We find a match between Quick Ride program and jumboization investment. As in Quick Ride program, capacities of the replenishment oilers are increased and a great deal of travel time are saved. As a bottom line, we convey that jumboization problem could be modeled with the consideration that expansion of capacity decrease the number of voyages and amount of time spent during voyages.

Jumboization investments in the U.S. Navy can also be modeled in the way that enlargement of a replenishment oiler defers purchasing of a new replenishment oiler. Moreover, another motivation for jumboization would be to avoid the risk of unserved demand.

Risk-Averse Decision Makers

Note that the frameworks we developed in main chapters of this dissertation are for risk-neutral decision makers. For instance, the probabilities of branches in lattice models which are used in backward recursion relations are risk-neutral probabilities. If decision makers are risk-averse, these frameworks are not applicable, which can be regarded as a limitation of our frameworks. Instead of using lattice models, one can use other decision analysis frameworks such as decision tree and utility theory under the consideration that decision makers are risk-averse.

Another point is that the U.S. Navy is too large organization to follow risk-averse approach in decision making. It implies that risk faced by the U.S. Navy can easily be diversified in many investments so that they do not have to be risk-averse in a single decision.

L_{ij} in OPF Problems

In this subsection, we will address an issue arisen for mathematical formulations of OPF problems. Particularly, in Equation (2A.4) of Chapter 2 and in Equation (4.17) of Chapter 4, the amount of power, denoted by L_{ij}, flowing on a power line between centers i and j is not a decision variable. Instead, it is calculated depending on the values of decision variables θ_i and θ_j, which denote voltage angles in centers i and j. One can see that L_{ij} is also accepted as a decision variable in some formulations of OPF problem. Indeed, this is not a requirement as it becomes a redundant decision variable.

Variance in Electricity Generation of DGs

Note that the framework we developed in Chapter 4 inherently assumes that a DG always produces electricity at its capacity when it is installed. In this chapter, we do not consider the variance in the amount of electricity generated by DGs. Our main focus is to model the uncertainty stemming from random installations or removals of DGs.

Different Stochastic Processes to Model the Evolution of Demand for Fuel

Note that in Chapter 3, we assume that demand for fuel by the receiving ships follows GBM process. We conduct statistical tests on a unique dataset (recall that we do not have available data of demand amount transported by a single replenishment oiler; rather we have an aggregated dataset in which we can see total amount of fuel transported by all replenishment oilers in a year) and verify that the assumption is valid in this context.

A question might arise as to what would happen if a different stochastic process is used to model the evolution of demand. It is known that the advantage of utilizing GBM process is its analytically tractable property. That is, it often leads to closed-form solutions, which facilitate to derive strong managerial insights. Beware that other stochastic processes such as Ornstein–Uhlenbeck process do not lead to closed-form solutions. In this case, numerical approaches such lattice frameworks or Monte Carlo simulations should be followed. Depending on a single process, GBM, can be counted as a weakness of our framework.

A Numerical Study on Computationally Efficient Lattice Framework Proposed in Chapter 4

In this subsection, we aim to demonstrate the efficiency of the lattice framework we proposed in Chapter 4. Recall that model 1 is the lattice model in which branches representing jump movements are drawn in each period. On the other hand, model 2 is the lattice model in which jump branches are drawn at every v periods. Our claim is model 2 approximates to model 1 given that λ, arrival rate of jump events, is sufficiently small. Hence, model 2 can be used instead of model 1 because we claim that value of investment at present time will approximate to each other in both models.

Let's revisit the numerical example solved in Chapter 4. All problem parameters are kept constant except the values of λ_1 and λ_3 are changed to 0.2 per year. Table 5.2 summarizes the results of computational study. Recall that $V_{(1,1)}$ denotes the value of transmission network at present time. We use Matlab to create both lattice models and conduct backward induction. We use fmincon function in Matlab to solve OPF problems.

Table 5.2 *Network values and computational times of models 1 and 2*

T	Model 1		Model 2	
	$V_{(1,1)}$ for base case ($)	Computation Time (seconds)	$V_{(1,1)}$ for base case ($)	Computation Time (seconds)
5	22,161,777	7910	22,097,376	107
4	17,572,582	436	17,449,836	27
3	12,620,269	27	12,490,514	7
2	8,248,654	2	8,248,654	2

As can be seen in Table 5.2, our proposed framework is able to obtain nearly the same transmission network values by saving a great deal of computation time.

CHAPTER 6. GENERAL CONCLUSION

In this dissertation, modeling aspects of specific problems arising in electric power transmission and fuel transportation areas are emphasized. The way of how economic decision making subject to real-life physical constraints can be followed is shown. In electric power transmission, these constraints are accepted as Kirchhoff current and voltage laws. As for fuel transportation in the navy, the relationships between speed, power, length and mass of the vehicle are considered as constraints.

In Chapter 2, for an electric power transmission problem, it is considered that the decision maker has the option to expand the network at any time through the modeling horizon. We show how physical laws of electricity can be utilized for determining local electricity prices, which determine the future revenue of a transmission investment. We also reflect that linear and much simpler OPF equations can be employed under certain conditions. This study reveals that the proportion of susceptance of a transmission line to its power carrying capacity affects the value of investment.

In Chapter 3, for a fuel transportation problem, it is accepted that the decision maker has the option to lengthen the transportation ship while it is in service. The value of lengthening the ship is quantified and a managerial guideline is provided regarding the choice between flexible and fixed designs. It reveals that relatively low level of transportation requirement at time zero is a signal for the decision maker to adopt the fixed design.

Due to lack of discrete disruptions in uncertain paths in Chapters 2 and 3, we study transmission expansion problem in Chapter 4 by considering both demand and DG installation uncertainties. The way of modeling those uncertainties in a unifying lattice model is shown. Because the computational complexity of the proposed model is significant, we propose an improvement idea to reduce the computation time. We use physical laws of electricity flow to determine the electricity prices at centers of transmission network. This study uncovers a significant managerial insight that installations of DGs do not necessarily lead to a reduction in the value of transmission network. The locations of installations play a key role to determine if a reduction happens. If an installed DG is in a consumption center which has a significant contribution to the calculation of LMPs (most probably due to inexistence of other generation units), the installation likely decreases the value of transmission network. If DG is installed in a consumption center which already possesses a generation unit, it is not likely to observe that DG undervalues transmission network.

Discrete disruptions exist in transportation requirements for fuel as well. For instance, if the U.S. Navy participates a training or a real war at sea (it happened in 2011 for Libya operations), the ships require much more fuel and this increases massively the amount of fuel transported by transportation vehicles. Therefore, the study presented in Chapter 4 can be extended in the way that fuel transportation requirement follows smooth changes as well as abrupt changes at random times.

This dissertation is created around three commonalities. Main chapters, Chapter 2, Chapter 3, and Chapter 4, all share the following aspects. First, the type of real option that we consider is expansion option. We consider that transmission networks can be expanded by adding a power line between two centers and the capacity of a ship can be expanded by extending its length by inserting a new mid-section. Secondly, the problems we study arise in the same industry, which is energy transportation sector. Electric power transferred by transmission lines and fuel carried by replenishment oilers are special types of energy commodities. Lastly, we use the same approach to model decision making frameworks in main chapters. We use real options (stochastic optimal control) to quantify the values of investments and values of options.

As a summary, this dissertation handles with an important problem in investment valuations. In real life, investment valuations are performed under critical physical constraints and significant uncertainties. To address this issue, we study transmission expansion planning and ship design problems in which Kirchoff laws and the relations between ship design parameters arise as physical constraints, respectively. Furthermore, we address both smooth changes and discrete disruptions in underlying uncertain parameters. We hope that this dissertation enlightens many aspects of questionable problems and leads to more plentiful studies.

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